You can use algebraic expressions and equations to model and analyze real-world situations. In this unit, you will learn about expressions, equations, and graphs.

Chapter 1
The Language of Algebra

Chapter 2
Real Numbers

Chapter 3
Solving Linear Equations
Then continue working on your WebQuest as you study Unit 1.

“...lost by the year 2020 as 100th birthdays become commonplace, predicts Mike Parker, assistant professor of social work, University of Alabama, Tuscaloosa, and a gerontologist specializing in successful aging. He says that, in the 21st century, the fastest growing age group in the country will be centenarians—those who live 100 years or longer.” In this project, you will explore how equations, functions, and graphs can help represent aging and population growth.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.
The Language of Algebra

**What You’ll Learn**

- **Lesson 1-1** Write algebraic expressions.
- **Lessons 1-2 and 1-3** Evaluate expressions and solve open sentences.
- **Lessons 1-4 through 1-6** Use algebraic properties of identity and equality.
- **Lesson 1-7** Use conditional statements and counterexamples.
- **Lessons 1-8 and 1-9** Interpret graphs of functions and analyze data in statistical graphs.

**Key Vocabulary**

- variable (p. 6)
- order of operations (p. 11)
- identity (p. 21)
- like terms (p. 28)
- counterexample (p. 38)

**Why It’s Important**

In every state and in every country, you find unique and inspiring architecture. Architects can use algebraic expressions to describe the volume of the structures they design. A few of the shapes these buildings can resemble are a rectangle, a pentagon, or even a pyramid. You will find the amount of space occupied by a pyramid in Lesson 1-2.
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1, 1-2, and 1-3

Multiply and Divide Whole Numbers
Find each product or quotient.

1. \(8 \times 8\)  
2. \(4 \times 16\)  
3. \(18 \div 9\)  
4. \(23 \div 6\)  
5. \(57 \div 3\)  
6. \(68 \div 4\)

For Lessons 1-1, 1-2, 1-5, and 1-6

Find Perimeter
(For review, see pages 813 and 814.)

9. Perimeter of a rectangle with sides 5.6 m and 2.7 m.
10. Perimeter of a rectangle with sides 6.5 cm and 3.05 cm.
11. Perimeter of a square with side 1\(\frac{3}{8}\) ft.
12. Perimeter of a rectangle with sides 42\(\frac{5}{8}\) ft and 25\(\frac{1}{4}\) ft.

For Lessons 1-5 and 1-6

Multiply and Divide Decimals and Fractions
Find each product or quotient. (For review, see pages 800 and 801.)

13. \(6 \times 1.2\)  
14. \(0.5 \times 3.9\)  
15. \(3.24 \div 1.8\)  
16. \(10.64 \div 1.4\)  
17. \(\frac{3}{4} \times 12\)  
18. \(1\frac{2}{3} \times \frac{3}{4}\)  
19. \(\frac{5}{16} \div \frac{9}{12}\)  
20. \(\frac{5}{6} \div \frac{2}{3}\)

Algebraic Properties
Make this Foldable to help you organize your notes. Begin with a sheet of notebook paper.

1. Fold
   Fold lengthwise to the holes.

2. Cut
   Cut along the top line and then cut 9 tabs.

3. Label
   Label the tabs using the lesson numbers and concepts.

Reading and Writing
Store the Foldable in a 3-ring binder. As you read and study the chapter, write notes and examples under the tabs.
WRITE MATHEMATICAL EXPRESSIONS In the algebraic expression $4s$, the letter $s$ is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable. *The letter $s$ was used above because it is the first letter of the word side.*

An **algebraic expression** consists of one or more numbers and variables along with one or more arithmetic operations. Here are some examples of algebraic expressions.

$$5x, \quad 3x - 7, \quad 4 + \frac{p}{q}, \quad m \times 5n, \quad 3ab + 5cd$$

In algebraic expressions, a raised dot or parentheses are often used to indicate multiplication as the symbol $\times$ can be easily mistaken for the letter $x$. Here are several ways to represent the product of $x$ and $y$.

$$xy, \quad x \cdot y, \quad x(y), \quad (x)y, \quad (x)(y)$$

In each expression, the quantities being multiplied are called **factors**, and the result is called the **product**.

It is often necessary to translate verbal expressions into algebraic expressions.

**Example 1** Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. **eight more than a number $n$**

The words *more than* suggest addition.

$$\frac{\text{eight}}{8} + \frac{\text{more than}}{n} \quad \frac{\text{a number } n}{n}$$

Thus, the algebraic expression is $8 + n$. 
b. 7 less than the product of 4 and a number $x$

*Less* implies subtract, and *product* implies multiply. So the expression can be written as $7 - 4x$.

c. one third of the size of the original area $a$

The word *of* implies multiply, so the expression can be written as $\frac{1}{3}a$ or $\frac{a}{3}$.

An expression like $x^n$ is called a **power** and is read “$x$ to the $n$th power.” The variable $x$ is called the **base**, and $n$ is called the **exponent**. The exponent indicates the number of times the base is used as a factor.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Words</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^1$</td>
<td>3 to the first power</td>
<td>3</td>
</tr>
<tr>
<td>$3^2$</td>
<td>3 to the second power or 3 squared</td>
<td>$3 \cdot 3$</td>
</tr>
<tr>
<td>$3^3$</td>
<td>3 to the third power or 3 cubed</td>
<td>$3 \cdot 3 \cdot 3$</td>
</tr>
<tr>
<td>$3^4$</td>
<td>3 to the fourth power</td>
<td>$3 \cdot 3 \cdot 3 \cdot 3$</td>
</tr>
<tr>
<td>$2b^6$</td>
<td>2 times $b$ to the sixth power</td>
<td>$2 \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$x$ to the $n$th power</td>
<td>$x \cdot x \cdot x \cdots x$ (n factors)</td>
</tr>
</tbody>
</table>

By definition, for any nonzero number $x$, $x^0 = 1$.

**Example 2** Write Algebraic Expressions with Powers

Write each expression algebraically.

<table>
<thead>
<tr>
<th>a. the product of 7 and $m$ to the fifth power</th>
<th>b. the difference of 4 and $x$ squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7m^5$</td>
<td>$4 - x^2$</td>
</tr>
</tbody>
</table>

To **evaluate** an expression means to find its value.

**Example 3** Evaluate Powers

Evaluate each expression.

<table>
<thead>
<tr>
<th>a. $2^6$</th>
<th>b. $4^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Use 2 as a factor 6 times.</td>
<td>$4^3 = 4 \cdot 4 \cdot 4$ Use 4 as a factor 3 times.</td>
</tr>
<tr>
<td>= 64 Multiply.</td>
<td>= 64 Multiply.</td>
</tr>
</tbody>
</table>

**WRITE VERBAL EXPRESSIONS** Another important skill is translating algebraic expressions into verbal expressions.

**Example 4** Write Verbal Expressions

Write a verbal expression for each algebraic expression.

<table>
<thead>
<tr>
<th>a. $4m^3$</th>
<th>b. $c^2 + 21d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>the product of 4 and $m$ to the third power</td>
<td>the sum of $c$ squared and 21 times $d$</td>
</tr>
</tbody>
</table>
Check for Understanding

**Concept Check**
1. Explain the difference between an algebraic expression and a verbal expression.
2. Write an expression that represents the perimeter of the rectangle.
3. OPEN ENDED Give an example of a variable to the fifth power.

**Guided Practice**
Write an algebraic expression for each verbal expression.
4. the sum of \( j \) and 13
5. 24 less than three times a number

Evaluate each expression.
6. \( 9^2 \)
7. \( 4^4 \)

Write a verbal expression for each algebraic expression.
8. \( 4m^4 \)
9. \( \frac{1}{2}n^3 \)

**Application**
10. MONEY Lorenzo bought several pounds of chocolate-covered peanuts and gave the cashier a $20 bill. Write an expression for the amount of change he will receive if \( p \) represents the cost of the peanuts.

Practice and Apply
Write an algebraic expression for each verbal expression.
11. the sum of 35 and \( z \)
12. the sum of a number and 7
13. the product of 16 and \( p \)
14. the product of 5 and a number
15. 49 increased by twice a number
16. 18 and three times \( d \)
17. two-thirds the square of a number
18. one-half the cube of \( n \)
19. SAVINGS Kendra is saving to buy a new computer. Write an expression to represent the amount of money she will have if she has \( s \) dollars saved and she adds \( d \) dollars per week for the next 12 weeks.
20. GEOMETRY The area of a circle can be found by multiplying the number \( \pi \) by the square of the radius. If the radius of a circle is \( r \), write an expression that represents the area of the circle.

Evaluate each expression.
21. \( 6^2 \)
22. \( 8^2 \)
23. \( 3^4 \)
24. \( 6^3 \)
25. \( 3^5 \)
26. \( 15^3 \)
27. \( 10^6 \)
28. \( 100^3 \)
29. FOOD A bakery sells a dozen bagels for $8.50 and a dozen donuts for $3.99. Write an expression for the cost of buying \( b \) dozen bagels and \( d \) dozen donuts.
30. **TRAVEL** Before starting her vacation, Sari’s car had 23,500 miles on the odometer. She drives an average of \( m \) miles each day for two weeks. Write an expression that represents the mileage on Sari’s odometer after her trip.

Write a verbal expression for each algebraic expression.

31. \( 7p \)  
32. \( 15r \)  
33. \( 3^3 \)  
34. \( 5^4 \)  
35. \( 3x^2 + 4 \)  
36. \( 2n^3 + 12 \)  
37. \( a^4 \cdot b^2 \)  
38. \( n^3 \cdot p^5 \)  
39. \( \frac{12c^2}{5} \)  
40. \( \frac{8d^3}{4} \)  
41. \( 3x^2 - 2x \)  
42. \( 4f^5 - 9k^3 \)

43. **PHYSICAL SCIENCE** When water freezes, its volume increases. The volume of ice equals the sum of the volume of the water and the product of one-eleventh and the volume of the water. If \( x \) cubic centimeters of water is frozen, write an expression for the volume of the ice that is formed.

44. **GEOMETRY** The surface area of a rectangular prism is the sum of:

- the product of twice the length \( \ell \) and the width \( w \),
- the product of twice the length and the height \( h \), and
- the product of twice the width and the height.

Write an expression that represents the surface area of a prism.

45. **RECYCLING** Each person in the United States produces approximately 3.5 pounds of trash each day. Write an expression representing the pounds of trash produced in a day by a family that has \( m \) members.  

Source: Vitality

46. **CRITICAL THINKING** In the square, the variable \( a \) represents a positive whole number. Find the value of \( a \) such that the area and the perimeter of the square are the same.

47. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

What expression can be used to find the perimeter of a baseball diamond? Include the following in your answer:

- two different verbal expressions that you can use to describe the perimeter of a square, and
- an algebraic expression other than \( 4s \) that you can use to represent the perimeter of a square.

48. What is 6 more than 2 times a certain number \( x \)?

\[ \text{A} \quad 2x - 6 \hspace{1cm} \text{B} \quad 2x \hspace{1cm} \text{C} \quad 6x - 2 \hspace{1cm} \text{D} \quad 2x + 6 \]

49. Write \( 4 \cdot 4 \cdot 4 \cdot c \cdot c \cdot c \) using exponents.

\[ \text{A} \quad 3^4c^3 \hspace{1cm} \text{B} \quad 4^3c^4 \hspace{1cm} \text{C} \quad (4c)^7 \hspace{1cm} \text{D} \quad 4c \]

**Maintain Your Skills**

Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Evaluate each expression.

(To review operations with fractions, see pages 798–801.)

50. \( 14.3 + 1.8 \)  
51. \( 10 - 3.24 \)  
52. \( 1.04 \times 4.3 \)  
53. \( 15.36 \div 4.8 \)  
54. \( \frac{1}{3} + \frac{2}{5} \)  
55. \( \frac{3}{4} - \frac{1}{6} \)  
56. \( \frac{3}{8} \times \frac{4}{9} \)  
57. \( \frac{7}{10} + \frac{3}{5} \)

Source: Vitality

Source: U.S. Environmental Protection Agency
You learned in Lesson 1-1 that it is often necessary to translate words into algebraic expressions. Generally, there are “clue” words such as *more than*, *times*, *less than*, and so on, which indicate the operation to use. These words also help to connect numerical data. The table shows a few examples.

<table>
<thead>
<tr>
<th>Words</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>four times $x$ plus $y$</td>
<td>$4x + y$</td>
</tr>
<tr>
<td>four times the sum of $x$ and $y$</td>
<td>$4(x + y)$</td>
</tr>
<tr>
<td>four times the quantity $x$ plus $y$</td>
<td>$4(x + y)$</td>
</tr>
</tbody>
</table>

Notice that all three expressions are worded differently, but the first expression is the only one that is different algebraically. In the second expression, parentheses indicate that the sum, $x + y$, is multiplied by four. In algebraic expressions, terms grouped by parentheses are treated as one quantity. So, $4(x + y)$ can also be read as *four times the quantity $x$ plus $y$*.

Words that may indicate parentheses are *sum*, *difference*, *product*, and *quantity*.

**Reading to Learn**

Read each verbal expression aloud. Then match it with the correct algebraic expression.

1. nine divided by 2 plus $n$
2. four divided by the difference of $n$ and six
3. $n$ plus five squared
4. three times the quantity eight plus $n$
5. nine divided by the quantity 2 plus $n$
6. three times eight plus $n$
7. the quantity $n$ plus five squared
8. four divided by $n$ minus six

<table>
<thead>
<tr>
<th>a. $(n + 5)^2$</th>
<th>b. $4 \div (n - 6)$</th>
<th>c. $9 \div 2 + n$</th>
<th>d. $3(8) + n$</th>
<th>e. $4 \div n - 6$</th>
<th>f. $n + 5^2$</th>
<th>g. $9 \div (2 + n)$</th>
<th>h. $3(8 + n)$</th>
</tr>
</thead>
</table>

Write each algebraic expression in words.

9. $5x + 1$
10. $5(x + 1)$
11. $3 + 7x$
12. $(3 + x) \cdot 7$
13. $(6 + b) \div y$
14. $6 \div (b \div y)$
**Vocabulary**

- order of operations

**What You’ll Learn**

- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

**How is the monthly cost of internet service determined?**

Nicole is signing up with a new internet service provider. The service costs $4.95 a month, which includes 100 hours of access. If she is online for more than 100 hours, she must pay an additional $0.99 per hour. Suppose Nicole is online for 117 hours the first month. The expression $4.95 + 0.99(117 - 100)$ represents what Nicole must pay for the month.

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**EVALUATE RATIONAL EXPRESSIONS**

Numerical expressions often contain more than one operation. A rule is needed to let you know which operation to perform first. This rule is called the **order of operations**.

**Key Concept**

**Order of Operations**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Evaluate expressions inside grouping symbols.</td>
</tr>
<tr>
<td>Step 2</td>
<td>Evaluate all powers.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Do all multiplications and/or divisions from left to right.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Do all additions and/or subtractions from left to right.</td>
</tr>
</tbody>
</table>

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**Example 1**

**Evaluate Expressions**

Evaluate each expression.

a. $3 + 2 \cdot 3 + 5$

$$3 + 2 \cdot 3 + 5 = 3 + 6 + 5 \quad \text{Multiply 2 and 3.}$$

$$= 9 + 5 \quad \text{Add 3 and 6.}$$

$$= 14 \quad \text{Add 9 and 5.}$$

b. $15 \div 3 \cdot 5 - 4^2$

$$15 \div 3 \cdot 5 - 4^2 = 15 \div 3 \cdot 5 - 16 \quad \text{Evaluate powers.}$$

$$= 5 \cdot 5 - 16 \quad \text{Divide 15 by 3.}$$

$$= 25 - 16 \quad \text{Multiply 5 by 5.}$$

$$= 9 \quad \text{Subtract 16 from 25.}$$
Grouping symbols such as parentheses ( ), brackets [ ], and braces { } are used to clarify or change the order of operations. They indicate that the expression within the grouping symbol is to be evaluated first.

Example 2  Grouping Symbols

Evaluate each expression.

a. \(2(5) + 3(4 + 3)\)

Evaluate inside grouping symbols.

\[\begin{align*}
2(5) + 3(4 + 3) &= 2(5) + 3(7) \\
&= 10 + 21 \\
&= 31
\end{align*}\]

Multiply expressions left to right.

Add 10 and 21.

b. \(2[5 + (30 \div 6)^2]\)

Evaluate innermost expression first.

\[\begin{align*}
2[5 + (30 \div 6)^2] &= 2[5 + (5)^2] \\
&= 2[5 + 25] \\
&= 2[30] \\
&= 60
\end{align*}\]

Evaluate expression in grouping symbol.

Multiply.

A fraction bar is another type of grouping symbol. It indicates that the numerator and denominator should each be treated as a single value.

Example 3  Fraction Bar

Evaluate \(\frac{6 + 4^2}{3^2 \cdot 4}\).

\(\frac{6 + 4^2}{3^2 \cdot 4}\) means \((6 + 4^2) \div (3^2 \cdot 4)\).

\[\begin{align*}
6 + 4^2 &= 6 + 16 \\
&= \frac{22}{3^2 \cdot 4} \\
&= \frac{22}{9 \cdot 4} \\
&= \frac{22}{36} \text{ or } \frac{11}{18}
\end{align*}\]

Evaluate the power in the numerator.

Add 6 and 16 in the numerator.

Evaluate the power in the denominator.

Multiply 9 and 4 in the denominator. Then simplify.

EVALUATE ALGEBRAIC EXPRESSIONS  Like numerical expressions, algebraic expressions often contain more than one operation. Algebraic expressions can be evaluated when the values of the variables are known. First, replace the variables with their values. Then, find the value of the numerical expression using the order of operations.

Example 4  Evaluate an Algebraic Expression

Evaluate \(a^2 - (b^3 - 4c)\) if \(a = 7\), \(b = 3\), and \(c = 5\).

Replace \(a\) with 7, \(b\) with 3, and \(c\) with 5.

\[\begin{align*}
a^2 - (b^3 - 4c) &= 7^2 - (3^3 - 4 \cdot 5) \\
&= 7^2 - (27 - 4 \cdot 5) \\
&= 7^2 - (27 - 20) \\
&= 7^2 - 7 \\
&= 49 - 7 \\
&= 42
\end{align*}\]

Evaluate \(3^3\).

Multiply 4 and 5.

Subtract 20 from 27.

Evaluate \(7^2\).

Subtract.
Lesson 1-2 Order of Operations

1. Describe how to evaluate $8[6^2/h11002/1003(2/h11001/5)]/h11004/8/h11001/3$.

2. OPEN ENDED Write an expression involving division in which the first step in evaluating the expression is addition.

3. FIND THE ERROR Laurie and Chase are evaluating $3[4 + (27/3)]^2$.

Laurie

$3[4 + (27/3)]^2 = 3(4 + 9^2)$

$= 3(4 + 81)$

$= 3(85)$

$= 255$

Chase

$3[4 + (27/3)]^2 = 3(4 + 9)^2$

$= 3(13)^2$

$= 3(169)$

$= 507$

Who is correct? Explain your reasoning.

Guided Practice

Evaluate each expression.

4. $(4 + 6)^7$

5. $50 - (15 + 9)$

6. $29 - 3(9 - 4)$

7. $[7(2) - 4] + [9 + 8(4)]$

8. $(4 \cdot 3)^2 \cdot 5/9 + 3$  

Evaluate each expression if $g = 4$, $h = 6$, $j = 8$, and $k = 12$.

9. $3 + 2^3/5^2(4)$

10. $hk - gj$

11. $2k + gh^2 - j$

12. $28(h - g)/gh - j$

Application SHOPPING For Exercises 13 and 14, use the following information.

A computer store has certain software on sale at 3 for $20.00, with a limit of 3 at the sale price. Additional software is available at the regular price of $9.95 each.

13. Write an expression you could use to find the cost of 5 software packages.

14. How much would 5 software packages cost?
Evaluate each expression.

15. \((12 - 6) \cdot 2\)
16. \((16 - 3) \cdot 4\)
17. \(15 + 3 \cdot 2\)
18. \(22 + 3 \cdot 7\)
19. \(4(11 + 7) - 9 \cdot 8\)
20. \(12(9 + 5) - 6 \cdot 3\)
21. \(12 \div 3 \cdot 5 - 4^2\)
22. \(15 \div 3 \cdot 5 - 4^2\)
23. \(288 \div [3(9 + 3)]\)

Evaluate each expression.

24. \(390 \div [5(7 + 6)]\)
25. \(\frac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}\)
26. \(\frac{4 \cdot 6^2 - 4^2 \cdot 6}{4 \cdot 6}\)
27. \(\frac{(8 + 5)(6 - 2)^2}{} - (4 \cdot 17 + 2)\)
28. \(\frac{(24 \div 2) + 3}{2}\)

29. GEOMETRY Find the area of the rectangle when \(n = 4\) centimeters.

ENTERTAINMENT For Exercises 30 and 31, use the following information.
Derrick and Samantha are selling tickets for their school musical. Floor seats cost $7.50 and balcony seats cost $5.00. Samantha sells 60 floor seats and 70 balcony seats, Derrick sells 50 floor seats and 90 balcony seats.

30. Write an expression to show how much money Samantha and Derrick have collected for tickets.

31. Evaluate the expression to determine how much they collected.

Evaluate each expression if \(x = 12\), \(y = 8\), and \(z = 3\).

32. \(x + y^2 + z^2\)
33. \(x^3 + y + z^3\)
34. \(3xy - z\)
35. \(4x - yz\)
36. \(\frac{2xy - z^3}{z}\)
37. \(\frac{xy^2 - 3z}{3}\)
38. \(\frac{x^2}{y} - \frac{3y - z}{(x - y)^2}\)
39. \(\frac{x - z^2}{y + x} + \frac{2y - x}{y^2 + 2}\)

40. BIOLOGY A certain type of bacteria can double its numbers every 20 minutes. Suppose 100 of these cells are in one culture dish and 250 of the cells are in another culture dish. Write and evaluate an expression that shows the total number of bacteria cells in both dishes after 20 minutes.

BUSINESS For Exercises 41–43, use the following information.
Mr. Martinez is a sales representative for an agricultural supply company. He receives a salary and monthly commission. He also receives a bonus each time he reaches a sales goal.

41. Write a verbal expression that describes how much Mr. Martinez earns in a year if he receives four equal bonuses.

42. Let \(e\) represent earnings, \(s\) represent his salary, \(c\) represent his commission, and \(b\) represent his bonus. Write an algebraic expression to represent his earnings if he receives four equal bonuses.

43. Suppose Mr. Martinez’s annual salary is $42,000 and his average commission is $825 each month. If he receives four bonuses of $750 each, how much does he earn in a year?
44. **CRITICAL THINKING** Choose three numbers from 1 to 6. Write as many expressions as possible that have different results when they are evaluated. You must use all three numbers in each expression, and each can only be used once.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is the monthly cost of internet service determined?**

Include the following in your answer:
- an expression for the cost of service if Nicole has a coupon for $25 off her base rate for her first six months, and
- an explanation of the advantage of using an algebraic expression over making a table of possible monthly charges.

46. Find the perimeter of the triangle using the formula \( P = a + b + c \) if \( a = 10 \), \( b = 12 \), and \( c = 17 \).

   - A) 39 mm
   - B) 19.5 mm
   - C) 60 mm
   - D) 78 mm

47. Evaluate \((5 - 1)^3 + (11 - 2)^2 + (7 - 4)^3\).

   - A) 586
   - B) 172
   - C) 106
   - D) 39

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**Maintain Your Skills**

**Mixed Review**

Write an algebraic expression for each verbal expression. *(Lesson 1-1)*

51. the product of the third power of \( a \) and the fourth power of \( b \)
52. six less than three times the square of \( y \)
53. the sum of \( a \) and \( b \) increased by the quotient of \( b \) and \( a \)
54. four times the sum of \( r \) and \( s \) increased by twice the difference of \( r \) and \( s \)
55. triple the difference of 55 and the cube of \( w \)

Evaluate each expression. *(Lesson 1-1)*

56. \( 2^4 \)
57. \( 12^1 \)
58. \( 8^2 \)
59. \( 4^4 \)

Write a verbal expression for each algebraic expression. *(Lesson 1-1)*

60. \( 5n + \frac{n}{2} \)
61. \( q^2 - 12 \)
62. \( \frac{(x + 3)}{(x - 2)^2} \)
63. \( \frac{x^3}{9} \)

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**PREREQUISITE SKILL** Find the value of each expression. *(To review operations with decimals and fractions, see pages 798–801.)*

64. \( 0.5 - 0.0075 \)
65. \( 5.6 + 1.612 \)
66. \( 14.9968 \div 5.2 \)
67. \( 2.3(6.425) \)
68. \( 4\frac{1}{8} - 1\frac{1}{2} \)
69. \( \frac{3}{5} + 2\frac{5}{7} \)
70. \( \frac{5}{6} \cdot \frac{4}{5} \)
71. \( 8 \div \frac{2}{9} \)

Visit [www.algebra1.com/self_check_quiz](http://www.algebra1.com/self_check_quiz)
SOLVE EQUATIONS  A mathematical statement with one or more variables is called an open sentence. An open sentence is neither true nor false until the variables have been replaced by specific values. The process of finding a value for a variable that results in a true sentence is called solving the open sentence. This replacement value is called a solution of the open sentence. A sentence that contains an equals sign, =, is called an equation.

A set of numbers from which replacements for a variable may be chosen is called a replacement set. A set is a collection of objects or numbers. It is often shown using braces, {}, and is usually named by a capital letter. Each object or number in the set is called an element, or member. The solution set of an open sentence is the set of elements from the replacement set that make an open sentence true.

Example 1  Use a Replacement Set to Solve an Equation

Find the solution set for each equation if the replacement set is {3, 4, 5, 6, 7}.

a. $6n + 7 = 37$

Replace $n$ in $6n + 7 = 37$ with each value in the replacement set.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$6n + 7 = 37$</th>
<th>True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$6(3) + 7 \neq 37 \rightarrow 25 \neq 37$</td>
<td>false</td>
</tr>
<tr>
<td>4</td>
<td>$6(4) + 7 \neq 37 \rightarrow 31 \neq 37$</td>
<td>false</td>
</tr>
<tr>
<td>5</td>
<td>$6(5) + 7 \neq 37 \rightarrow 37 = 37$</td>
<td>true</td>
</tr>
<tr>
<td>6</td>
<td>$6(6) + 7 \neq 37 \rightarrow 43 \neq 37$</td>
<td>false</td>
</tr>
<tr>
<td>7</td>
<td>$6(7) + 7 \neq 37 \rightarrow 49 \neq 37$</td>
<td>false</td>
</tr>
</tbody>
</table>

Since $n = 5$ makes the equation true, the solution of $6n + 7 = 37$ is 5.
The solution set is {5}.
b. 5(x + 2) = 40

Replace x in 5(x + 2) = 40 with each value in the replacement set.

<table>
<thead>
<tr>
<th>x</th>
<th>5(x + 2) = 40</th>
<th>True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5(3 + 2) ≤ 40 → 25 ≠ 40</td>
<td>false</td>
</tr>
<tr>
<td>4</td>
<td>5(4 + 2) ≤ 40 → 30 ≠ 40</td>
<td>false</td>
</tr>
<tr>
<td>5</td>
<td>5(5 + 2) ≤ 40 → 35 ≠ 40</td>
<td>false</td>
</tr>
<tr>
<td>6</td>
<td>5(6 + 2) ≤ 40 → 40 = 40</td>
<td>true ✓</td>
</tr>
<tr>
<td>7</td>
<td>5(7 + 2) ≤ 40 → 45 ≠ 40</td>
<td>false</td>
</tr>
</tbody>
</table>

The solution of 5(x + 2) = 40 is 6. The solution set is \{6\}.

You can often solve an equation by applying the order of operations.

**Example 2** Use Order of Operations to Solve an Equation

Solve \(\frac{13 + 2(4)}{3(5 - 4)} = q\).

\[
\frac{13 + 2(4)}{3(5 - 4)} = q \quad \text{Original equation}
\]

\[
\frac{13 + 8}{3(1)} = q \quad \text{Multiply 2 and 4 in the numerator.}
\]

\[
\frac{21}{3} = q \quad \text{Subtract 4 from 5 in the denominator.}
\]

\[
7 = q \quad \text{Simplify.}
\]

\[
\text{Divide.} \quad \text{The solution is 7.}
\]

**Example 3** Find the Solution Set of an Inequality

Find the solution set for \(18 - y < 10\) if the replacement set is \{7, 8, 9, 10, 11, 12\}.

Replace y in \(18 - y < 10\) with each value in the replacement set.

<table>
<thead>
<tr>
<th>y</th>
<th>18 - y &lt; 10</th>
<th>True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>18 - 7 ≤ 10 → 11 ≤ 10</td>
<td>false</td>
</tr>
<tr>
<td>8</td>
<td>18 - 8 ≤ 10 → 10 ≤ 10</td>
<td>false</td>
</tr>
<tr>
<td>9</td>
<td>18 - 9 ≤ 10 → 9 ≤ 10</td>
<td>true ✓</td>
</tr>
<tr>
<td>10</td>
<td>18 - 10 ≤ 10 → 8 ≤ 10</td>
<td>true ✓</td>
</tr>
<tr>
<td>11</td>
<td>18 - 11 ≤ 10 → 7 ≤ 10</td>
<td>true ✓</td>
</tr>
<tr>
<td>12</td>
<td>18 - 12 ≤ 10 → 6 ≤ 10</td>
<td>true ✓</td>
</tr>
</tbody>
</table>

The solution set for \(18 - y < 10\) is \{9, 10, 11, 12\}.

**Example 4** Solve an Inequality

**FUND-RAISING** Refer to the application at the beginning of the lesson. How many garage sale kits can the association buy and stay within their budget?

**Explore** The association can spend no more than $135. So the situation can be represented by the inequality \(15.50 + 5n ≤ 135\).
Plan Since no replacement set is given, estimate to find reasonable values for the replacement set.

Solve Start by letting \( n = 10 \) and then adjust values up or down as needed.

\[
15.50 + 5n \leq 135 \quad \text{Original inequality}
\]
\[
15.50 + 5(10) \leq 135 \quad n = 10
\]
\[
15.50 + 50 \leq 135 \quad \text{Multiply 5 and 10.}
\]
\[
65.50 \leq 135 \quad \text{Add 15.50 and 50.}
\]

The estimate is too low. Increase the value of \( n \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 15.50 + 5n \leq 135 )</th>
<th>Reasonable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>( 15.50 + 5(20) \leq 135 \Rightarrow 115.50 \leq 135 )</td>
<td>too low</td>
</tr>
<tr>
<td>25</td>
<td>( 15.50 + 5(25) \leq 135 \Rightarrow 140.50 \leq 135 )</td>
<td>too high</td>
</tr>
<tr>
<td>23</td>
<td>( 15.50 + 5(23) \leq 135 \Rightarrow 130.50 \leq 135 )</td>
<td>almost</td>
</tr>
<tr>
<td>24</td>
<td>( 15.50 + 5(24) \leq 135 \Rightarrow 135.50 \leq 135 )</td>
<td>too high</td>
</tr>
</tbody>
</table>

Examine The solution set is \( \{0, 1, 2, 3, \ldots, 21, 22, 23\} \). In addition to the first kit, the association can buy as many as 23 additional kits. So, the association can buy as many as \( 1 + 23 \) or 24 garage sale kits and stay within their budget.

---

**Check for Understanding**

**Concept Check**

1. **Describe** the difference between an expression and an open sentence.

2. **OPEN ENDED** Write an inequality that has a solution set of \( \{8, 9, 10, 11, \ldots\} \).

3. **Explain** why an open sentence always has at least one variable.

**Guided Practice**

Find the solution of each equation if the replacement set is \( \{10, 11, 12, 13, 14, 15\} \).

4. \( 3x - 7 = 29 \)

5. \( 12(x - 8) = 84 \)

Find the solution of each equation using the given replacement set.

6. \( x + \frac{2}{5} = 1\frac{3}{20}; \left\{ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{4}\right\} \)

7. \( 7.2(x + 2) = 25.92; \{1.2, 1.4, 1.6, 1.8\} \)

Solve each equation.

8. \( 4(6) + 3 = x \)

9. \( w = \frac{14 - 8}{2} \)

Find the solution set for each inequality using the given replacement set.

10. \( 24 - 2x \geq 13; \{0, 1, 2, 3, 4, 5, 6\} \)

11. \( 3(12 - x) - 2 \leq 28; \{1.5, 2, 2.5, 3\} \)

**Application** **NUTRITION** For Exercises 12 and 13, use the following information.

A person must burn 3500 Calories to lose one pound of weight.

12. Write an equation that represents the number of Calories a person would have to burn a day to lose four pounds in two weeks.

13. How many Calories would the person have to burn each day?
Find the solution of each equation if the replacement sets are \( a = \{0, 3, 5, 8, 10\} \)
and \( b = \{12, 17, 18, 21, 25\} \).

14. \( b - 12 = 9 \)  
15. \( 34 - b = 22 \)  
16. \( 3a + 7 = 31 \)  
17. \( 4a + 5 = 17 \)  
18. \( \frac{40}{a} - 4 = 0 \)  
19. \( \frac{b}{3} - 2 = 4 \)

Find the solution of each equation using the given replacement set.

20. \( x + \frac{7}{4} = \frac{17}{8} \)  
21. \( x + \frac{7}{12} = \frac{25}{12}, \left\{ \frac{1}{2}, 1, \frac{1}{2}, 2 \right\} \)

22. \( \frac{2}{5}(x + 1) = \frac{8}{15}, \left\{ \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, 3 \right\} \)

23. \( 2.7(x + 5) = 17.28; \{1.2, 1.3, 1.4, 1.5\} \)

24. \( 16(x + 2) = 70.4; \{2.2, 2.4, 2.6, 2.8\} \)

25. \( 21(x + 5) = 216.3; \{3.1, 4.2, 5.3, 6.4\} \)

**MOVIES** For Exercises 26–28, use the table and the following information.
The Conkle family is planning to see a movie. There are two adults, a daughter in high school, and two sons in middle school. They do not want to spend more than $30.

26. The movie theater charges the same price for high school and middle school students. Write an inequality to show the cost for the family to go to the movies.

27. How much will it cost for the family to see a matinee?

28. How much will it cost to see an evening show?

**Admission Prices**

<table>
<thead>
<tr>
<th></th>
<th>Evening</th>
<th>Matinee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult</td>
<td>$7.50</td>
<td></td>
</tr>
<tr>
<td>Student</td>
<td>$4.50</td>
<td></td>
</tr>
<tr>
<td>Child</td>
<td>$3.50</td>
<td></td>
</tr>
<tr>
<td>Senior</td>
<td>$4.50</td>
<td></td>
</tr>
<tr>
<td>All Seats</td>
<td>$4.50</td>
<td></td>
</tr>
</tbody>
</table>

**Solve each equation.**

29. \( 14.8 - 3.75 = t \)
30. \( a = 32.4 - 18.95 \)
31. \( y = \frac{12 \cdot 5}{15 - 3} \)
32. \( g = \frac{15 \cdot 6}{16 - 7} \)
33. \( d = \frac{7(3) + 3}{4(3) - 1} + 6 \)
34. \( a = \frac{4(14 - 1)}{3(6) - 5} + 7 \)
35. \( p = \frac{1}{4}[7(2^3) + 4(5^2) - 6(2)] \)
36. \( n = \frac{1}{8}[6(3^2) + 2(4^3) - 2(7)] \)

**Find the solution set for each inequality using the given replacement set.**

37. \( a - 2 < 6; \{6, 7, 8, 9, 10, 11\} \)
38. \( a + 7 < 22; \{13, 14, 15, 16, 17\} \)
39. \( \frac{a}{5} \geq 2; \{5, 10, 15, 20, 25\} \)
40. \( \frac{2a}{4} \leq 8; \{12, 14, 16, 18, 20, 22\} \)
41. \( 4a - 3 \geq 10.6; \{3.2, 3.4, 3.6, 3.8, 4\} \)
42. \( 6a - 5 \geq 23.8; \{4.2, 4.5, 4.8, 5.1, 5.4\} \)
43. \( 3a \leq 4; \left\{ 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{1}{3} \right\} \)
44. \( 2b < 5; \left\{ 1, \frac{1}{2}, 2, 2\frac{1}{2}, 3 \right\} \)

**FOOD** For Exercises 45 and 46, use the information about food at the left.

45. Write an equation to find the total number of glasses of milk, juice, and soda the average American drinks in a lifetime.

46. How much milk, juice, and soda does the average American drink in a lifetime?

**MAIL ORDER** For Exercises 47 and 48, use the following information.
Suppose you want to order several sweaters that cost $39.00 each from an online catalog. There is a $10.95 charge for shipping. You have $102.50 to spend.

47. Write an inequality you could use to determine the maximum number of sweaters you can purchase.

48. What is the maximum number of sweaters you can buy?
49. **CRITICAL THINKING**  Describe the solution set for \( x \) if \( 3x \leq 1 \).

50. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

How can you use open sentences to stay within a budget?

Include the following in your answer:

- an explanation of how to use open sentences to stay within a budget, and
- examples of real-world situations in which you would use an inequality and examples where you would use an equation.

51. Find the solution set for \( \frac{5 \cdot n^2 + 5}{(9 \cdot 3^2) - n} < 28 \) if the replacement set is \{5, 7, 9, 11, 13\}.

- A \{5\}
- B \{5, 7\}
- C \{7\}
- D \{7, 9\}

52. Which expression has a value of 17?

- A \((9 \times 3) - 63 \div 7\)
- B \(6(3 + 2) + (9 - 7)\)
- C \(27 \div 3 + (12 - 4)\)
- D \(2[2(6 - 3)] - 5\)

---

**Maintain Your Skills**

**Mixed Review**  Write an algebraic expression for each verbal expression. Then evaluate each expression if \( r = 2 \), \( s = 5 \), and \( t = \frac{1}{2} \).  

(Lesson 1-2)

53. \( r \) squared increased by 3 times \( s \)

54. \( t \) times the sum of four times \( s \) and \( r \)

55. the sum of \( r \) and \( s \) times the square of \( t \)

56. \( r \) to the fifth power decreased by \( t \)

Evaluate each expression.  

(Lesson 1-2)

57. \( 5^3 + 3(4^2) \)  
58. \( \frac{38 - 12}{2 \cdot 13} \)

59. \([5(2 + 1)]^4 + 3\)

**Getting Ready for the Next Lesson**  Find each product. Express in simplest form.

(To review multiplying fractions, see pages 800 and 801)

(Lesson 1-2)

60. \( \frac{1}{6} \cdot \frac{2}{5} \)

61. \( \frac{4}{9} \cdot \frac{3}{7} \)

62. \( \frac{5}{6} \cdot \frac{15}{16} \)

63. \( \frac{6}{14} \cdot \frac{12}{18} \)

64. \( \frac{8}{13} \cdot \frac{2}{11} \)

65. \( \frac{4}{7} \cdot \frac{4}{9} \)

66. \( \frac{3}{11} \cdot \frac{7}{16} \)

67. \( \frac{2}{9} \cdot \frac{24}{25} \)

---

**Practice Quiz 1**  

(Lessons 1-1 through 1-3)

Write a verbal expression for each algebraic expression.  

(Lesson 1-1)

1. \( x - 20 \)
2. \( 5n + 2 \)
3. \( a^3 \)
4. \( n^4 - 1 \)

Evaluate each expression.  

(Lesson 1-2)

5. \( 6(9) - 2(8 + 5) \)
6. \( 4[2 + (18 + 9)^3] \)
7. \( 9(3) - 4^2 + 6^2 \div 2 \)
8. \( \frac{(5 - 2)^2}{3(4 \cdot 2 - 7)} \)

9. Evaluate \( \frac{5a^2 + c - 2}{6 + b} \) if \( a = 4 \), \( b = 5 \), and \( c = 10 \).  

(Lesson 1-2)

10. Find the solution set for \( 2n^2 + 3 \leq 75 \) if the replacement set is \{4, 5, 6, 7, 8, 9\}.  

(Lesson 1-3)
Identity and Equality Properties

What You’ll Learn

- Recognize the properties of identity and equality.
- Use the properties of identity and equality.

Vocabulary

- additive identity
- multiplicative identity
- multiplicative inverses
- reciprocal

How are identity and equality properties used to compare data?

During the college football season, teams are ranked weekly. The table shows the last three rankings of the top five teams for the 2000 football season. The open sentence below represents the change in rank of Oregon State from December 11 to the final rank.

<table>
<thead>
<tr>
<th>Rank on December 11, 2000</th>
<th>plus</th>
<th>increase in rank</th>
<th>equals</th>
<th>final rank for 2000 season</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>+</td>
<td>r</td>
<td>=</td>
<td>4</td>
</tr>
</tbody>
</table>

The solution of this equation is 0. Oregon State’s rank changed by 0 from December 11 to the final rank. In other words, $4 + 0 = 4$.

IDENTITY AND EQUALITY PROPERTIES

The sum of any number and 0 is equal to the number. Thus, 0 is called the additive identity.

**Key Concept**

**Additive Identity**

- **Words** For any number $a$, the sum of $a$ and 0 is $a$.
- **Symbols** $a + 0 = 0 + a = a$
- **Examples** $5 + 0 = 5, 0 + 5 = 5$

There are also special properties associated with multiplication. Consider the following equations.

$$7 \cdot n = 7$$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the multiplicative identity.

$$9 \cdot m = 0$$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

$$\frac{1}{3} \cdot 3 = 1$$

Two numbers whose product is 1 are called multiplicative inverses or reciprocals. Zero has no reciprocal because any number times 0 is 0.
### Key Concept

**Multiplication Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative Identity</td>
<td>For any number ( a ), the product of ( a ) and 1 is ( a ).</td>
<td>( a \cdot 1 = 1 \cdot a = a )</td>
<td>( 12 \cdot 1 = 12, \ 1 \cdot 12 = 12 )</td>
</tr>
<tr>
<td>Multiplicative Property of Zero</td>
<td>For any number ( a ), the product of ( a ) and 0 is 0.</td>
<td>( a \cdot 0 = 0 \cdot a = 0 )</td>
<td>( 8 \cdot 0 = 0, \ 0 \cdot 8 = 0 )</td>
</tr>
<tr>
<td>Multiplicative Inverse</td>
<td>For every number ( \frac{a}{b} ) where ( a, b \neq 0 ), there is exactly one number ( \frac{b}{a} ) such that the product of ( \frac{a}{b} ) and ( \frac{b}{a} ) is 1.</td>
<td>( \frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1 )</td>
<td>( \frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1 )</td>
</tr>
</tbody>
</table>

---

### Example 1

**Identify Properties**

Name the property used in each equation. Then find the value of \( n \).

a. \( 42 \cdot n = 42 \)
   - Multiplicative Identity Property
   - \( n = 1 \), since \( 42 \cdot 1 = 42 \).

b. \( n + 0 = 15 \)
   - Additive Identity Property
   - \( n = 15 \), since \( 15 + 0 = 15 \).

c. \( n \cdot 9 = 1 \)
   - Multiplicative Inverse Property
   - \( n = \frac{1}{9} \), since \( \frac{1}{9} \cdot 9 = 1 \).

There are several properties of equality that apply to addition and multiplication. These are summarized below.

### Key Concept

**Properties of Equality**

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>Any quantity is equal to itself.</td>
<td>For any number ( a ), ( a = a ).</td>
<td>( 7 = 7, \ 2 + 3 = 2 + 3 )</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If one quantity equals a second quantity, then the second quantity equals the first.</td>
<td>For any numbers ( a ) and ( b ), if ( a = b ), then ( b = a ).</td>
<td>If ( 9 = 6 + 3 ), then ( 6 + 3 = 9 ).</td>
</tr>
<tr>
<td>Transitive</td>
<td>If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.</td>
<td>For any numbers ( a, b ), and ( c ), if ( a = b ) and ( b = c ), then ( a = c ).</td>
<td>If ( 5 + 7 = 8 + 4 ) and ( 8 + 4 = 12 ), then ( 5 + 7 = 12 ).</td>
</tr>
<tr>
<td>Substitution</td>
<td>A quantity may be substituted for its equal in any expression.</td>
<td>If ( a = b ), then ( a ) may be replaced by ( b ) in any expression.</td>
<td>If ( n = 15 ), then ( 3n = 3 \cdot 15 ).</td>
</tr>
</tbody>
</table>
USE IDENTITY AND EQUALITY PROPERTIES  The properties of identity and equality can be used to justify each step when evaluating an expression.

Example 2  Evaluate Using Properties

Evaluate $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$. Name the property used in each step.

$2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3} = 2(6 - 5) + 3 \cdot \frac{1}{3}$  
Substitution; $3 \cdot 2 = 6$

$= 2(1) + 3 \cdot \frac{1}{3}$  
Substitution; $6 - 5 = 1$

$= 2 + 3 \cdot \frac{1}{3}$  
Multiplicative Identity; $2 \cdot 1 = 2$

$= 2 + 1$  
Multiplicative Inverse; $3 \cdot \frac{1}{3} = 1$

$= 3$  
Substitution; $2 + 1 = 3$

Check for Understanding

Concept Check
1. Explain whether 1 can be an additive identity.
2. OPEN ENDED Write two equations demonstrating the Transitive Property of Equality.
3. Explain why 0 has no multiplicative inverse.

Guided Practice
Name the property used in each equation. Then find the value of $n$.
4. $13n = 0$
5. $17 + 0 = n$
6. $\frac{1}{6}n = 1$

7. Evaluate $6(12 - 48 \div 4)$. Name the property used in each step.
8. Evaluate $\left(15 \cdot \frac{1}{15} + 8 \cdot 0\right) \cdot 12$. Name the property used in each step.

Application  HISTORY  For Exercises 9–11, use the following information.
On November 19, 1863, Abraham Lincoln delivered the famous Gettysburg Address. The speech began “Four score and seven years ago, . . .”

9. Write an expression to represent four score and seven. (Hint: A score is 20.)
10. Evaluate the expression. Name the property used in each step.
11. How many years is four score and seven?

Practice and Apply

Name the property used in each equation. Then find the value of $n$.
12. $12n = 12$
13. $n \cdot 1 = 5$
14. $8 \cdot n = 8 \cdot 5$
15. $0.25 + 1.5 = n + 1.5$
16. $8 = n + 8$
17. $n + 0 = \frac{1}{3}$
18. $1 = 2n$
19. $4 \cdot \frac{1}{4} = n$
20. $(9 - 7)(5) = 2(n)$
21. $3 + (2 + 8) = n + 10$
22. $n\left(5^2 \cdot \frac{1}{25}\right) = 3$
23. $6\left(\frac{1}{2} \cdot n\right) = 6$
24. $\frac{3}{4}[4 \div (7 - 4)]$
25. $\frac{2}{3}[3 \div (2 \cdot 1)]$
26. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$
27. $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$
28. $3 + 5(4 - 2^2) - 1$
29. $7 - 8(9 - 3^2)$

www.algebra1.com/extra_examples
**FUND-RAISING**  For Exercises 30 and 31, use the following information.
The spirit club at Marshall High School is selling items to raise money. The profit the club earns on each item is the difference between what an item sells for and what it costs the club to buy.

30. Write an expression that represents the profit for 25 pennants, 80 buttons, and 40 caps.

31. Evaluate the expression, indicating the property used in each step.

**MILITARY PAY**  For Exercises 32 and 33, use the table below.

<table>
<thead>
<tr>
<th>Enlisted Personnel Monthly Pay Rates, Effective July 1, 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>E-5</td>
</tr>
<tr>
<td>E-4</td>
</tr>
<tr>
<td>E-3</td>
</tr>
<tr>
<td>E-2</td>
</tr>
<tr>
<td>E-1</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Defense

32. Write an equation using addition that shows the change in pay for an enlisted member at grade E-2 from 3 years of service to 12 years.

33. Write an equation using multiplication that shows the change in pay for someone at grade E-4 from 6 years of service to 10 years.

**FOOTBALL**  For Exercises 34–36, use the table of base salaries and bonus plans.

34. Suppose a player rushed for 12 touchdowns in 2002 and another player scored 76 points that same year. Write an equation that compares the two salaries and bonuses.

35. Write an expression that could be used to determine the base salaries and bonuses in 2004 for the following:

- eight players who keep their weight under 240 pounds and are involved in at least 35% of the offensive plays,
- three players who score 12 rushing touchdowns and score 76 points, and
- four players who gain 1601 yards of total offense and average 4.5 yards per carry.

36. Evaluate the expression you wrote in Exercise 35. Name the property used in each step.

**Online Research Data Update** Find the most recent statistics for a professional football player. What were his base salary and bonuses? Visit www.algebra1.com/data_update to learn more.
37. **CRITICAL THINKING**  The Transitive Property of Inequality states that if \( a < b \) and \( b < c \), then \( a < c \). Use this property to determine whether the following statement is sometimes, always, or never true.

\[
\text{If } x > y \text{ and } z > w, \text{ then } xz > yw.
\]

Give examples to support your answer.

38. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

How are identity and equality properties used to compare data?

Include the following in your answer:

- a description of how you could use the Reflexive or Symmetric Property to compare a team’s rank for any two time periods, and
- a demonstration of the Transitive Property using one of the team’s three rankings as an example.

39. Which equation illustrates the Symmetric Property of Equality?

- **A** If \( a = b \), then \( b = a \).
- **B** If \( a = b = c \), then \( a = c \).
- **C** If \( a = b \), then \( b = c \).
- **D** If \( a = a \), then \( a + 0 = a \).

40. The equation \((10 - 8)(5) = (2)(5)\) is an example of which property of equality?

- **A** Reflexive
- **B** Substitution
- **C** Symmetric
- **D** Transitive

The sum of any two whole numbers is always a whole number. So, the set of whole numbers \( \{0, 1, 2, 3, \ldots \} \) is said to be closed under addition. This is an example of the **Closure Property**. State whether each of the following statements is true or false. If false, justify your reasoning.

41. The set of whole numbers is closed under subtraction.
42. The set of whole numbers is closed under multiplication.
43. The set of whole numbers is closed under division.

### Maintain Your Skills

**Mixed Review**  Find the solution set for each inequality using the given replacement set.  *(Lesson 1-3)*

44. \( 10 - x > 6; \{3, 5, 6, 8\} \)
45. \( 4x + 2 < 58; \{11, 12, 13, 14, 15\} \)
46. \( \frac{x}{2} \geq 3; \{5.8, 5.9, 6, 6.1, 6.2, 6.3\} \)
47. \( 8x \leq 32; \{3, 3.25, 3.5, 3.75, 4\} \)
48. \( \frac{7}{10} - 2x < \frac{3}{10}; \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \right\} \)
49. \( 2x - 1 \leq 2; \left\{ \frac{1}{14}, 2, 3, 3\frac{1}{2} \right\} \)

**Evaluate each expression.**  *(Lesson 1-2)*

50. \( (3 + 6) \div 3^2 \)
51. \( 6(12 - 7.5) - 7 \)
52. \( 20 \div 4 \cdot 8 + 10 \)
53. \( \frac{(6 + 2)^2}{16} + 3(9) \)
54. \( [6^2 - (2 + 4)]3 \)
55. \( 9(3) - 4^2 + 6^2 \div 2 \)

56. Write an algebraic expression for the sum of twice a number squared and 7.  *(Lesson 1-1)*

**Getting Ready for the Next Lesson**  **PREREQUISITE SKILL**  Evaluate each expression.  *(To review order of operations, see Lesson 1-2)*

57. \( 10(6) + 10(2) \)
58. \( (15 - 6) \cdot 8 \)
59. \( 12(4) - 5(4) \)
60. \( 3(4 + 2) \)
61. \( 5(6 - 4) \)
62. \( 8(14 + 2) \)

www.algebra1.com/self_check_quiz
The Distributive Property

What You’ll Learn

• Use the Distributive Property to evaluate expressions.
• Use the Distributive Property to simplify expressions.

Vocabulary

• term
• like terms
• equivalent expressions
• simplest form
• coefficient

How can the Distributive Property be used to calculate quickly?

Instant Replay Video Games sells new and used games. During a Saturday morning sale, the first 8 customers each bought a bargain game and a new release. To calculate the total sales for these customers, you can use the Distributive Property.

<table>
<thead>
<tr>
<th>Sale Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Games</td>
</tr>
<tr>
<td>Bargain Games</td>
</tr>
<tr>
<td>Regular Games</td>
</tr>
<tr>
<td>New Releases</td>
</tr>
</tbody>
</table>

EVALUATE EXPRESSIONS

There are two methods you could use to calculate the video game sales.

**Method 1**

<table>
<thead>
<tr>
<th>sales of bargain games</th>
<th>plus</th>
<th>sales of new releases</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(14.95)</td>
<td>+</td>
<td>8(34.95)</td>
</tr>
</tbody>
</table>

= 119.60 + 279.60
= 399.20

**Method 2**

<table>
<thead>
<tr>
<th>number of customers</th>
<th>times</th>
<th>each customer’s purchase price</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>×</td>
<td>(14.95 + 34.95)</td>
</tr>
</tbody>
</table>

= 8(49.90)
= 399.20

Either method gives total sales of $399.20 because the following is true.

8(14.95) + 8(34.95) = 8(14.95 + 34.95)

This is an example of the Distributive Property.

Key Concept

**Distributive Property**

- **Symbols**
  For any numbers a, b, and c,
  
  \[ a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca \]

- **Examples**
  
  \[ 3(2 + 5) = 3 \cdot 2 + 3 \cdot 5 \quad 4(9 - 7) = 4 \cdot 9 - 4 \cdot 7 \]
  
  \[ 3(7) = 6 + 15 \quad 4(2) = 36 - 28 \]
  
  \[ 21 = 21 \quad 8 = 8 \]

Notice that it does not matter whether a is placed on the right or the left of the expression in the parentheses.

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

\[ a(b + c) = ab + ac, \text{ then } ab + ac = a(b + c). \]
**Example 1 Distribute Over Addition**

Rewrite $8(10 + 4)$ using the Distributive Property. Then evaluate.

\[
8(10 + 4) = 8(10) + 8(4) \quad \text{Distributive Property}
\]

\[
= 80 + 32 \quad \text{Multiply.}
\]

\[
= 112 \quad \text{Add.}
\]

**Example 2 Distribute Over Subtraction**

Rewrite $(12 - 3)6$ using the Distributive Property. Then evaluate.

\[
(12 - 3)6 = 12 \cdot 6 - 3 \cdot 6 \quad \text{Distributive Property}
\]

\[
= 72 - 18 \quad \text{Multiply.}
\]

\[
= 54 \quad \text{Subtract.}
\]

**Example 3 Use the Distributive Property**

**CARS** The Morris family owns two cars. In 1998, they drove the first car 18,000 miles and the second car 16,000 miles. Use the graph to find the total cost of operating both cars.

Use the Distributive Property to write and evaluate an expression.

\[
0.46(18,000 + 16,000) \quad \text{Distributive Prop.}
\]

\[
= 8280 + 7360 \quad \text{Multiply.}
\]

\[
= 15,640 \quad \text{Add.}
\]

It cost the Morris family $15,640 to operate their cars.

The Distributive Property can be used to simplify mental calculations.

**Example 4 Use the Distributive Property**

Use the Distributive Property to find each product.

a. $15 \cdot 99$

\[
15 \cdot 99 = 15(100 - 1) \quad \text{Think: } 99 = 100 - 1
\]

\[
= 15(100) - 15(1) \quad \text{Distributive Property}
\]

\[
= 1500 - 15 \quad \text{Multiply.}
\]

\[
= 1485 \quad \text{Subtract.}
\]

b. $35\left(2\frac{1}{5}\right)$

\[
35\left(2\frac{1}{5}\right) = 35\left(2 + \frac{1}{5}\right) \quad \text{Think: } 2\frac{1}{5} = 2 + \frac{1}{5}
\]

\[
= 35(2) + 35\left(\frac{1}{5}\right) \quad \text{Distributive Property}
\]

\[
= 70 + 7 \quad \text{Multiply.}
\]

\[
= 77 \quad \text{Add.}
\]
**Example 5** Algebraic Expressions

Rewrite each product using the Distributive Property. Then simplify.

a. $5(g - 9)$

$$5(g - 9) = 5 \cdot g - 5 \cdot 9 \quad \text{Distributive Property}$$

$$= 5g - 45 \quad \text{Multiply.}$$

b. $3(2x^2 + 4x - 1)$

$$3(2x^2 + 4x - 1) = (3)(2x^2) + (3)(4x) - 3(1) \quad \text{Distributive Property}$$

$$= 6x^2 + 12x - 3 \quad \text{Simplify.}$$

A **term** is a number, a variable, or a product or quotient of numbers and variables. For example, $y$, $p^3$, $4a$, and $5g^2h$ are all terms. **Like terms** are terms that contain the same variables, with corresponding variables having the same power.

---

SIMPLIFY EXPRESSIONS You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.

**Algebra Activity**

The Distributive Property

Consider the product $3(x + 2)$. Use a product mat and algebra tiles to model $3(x + 2)$ as the area of a rectangle whose dimensions are 3 and $(x + 2)$.

**Step 1** Use algebra tiles to mark the dimensions of the rectangle on a product mat.

---

Model and Analyze

Find each product by using algebra tiles.

1. $2(x + 1)$
2. $5(x + 2)$
3. $2(2x + 1)$

Tell whether each statement is **true** or **false**. Justify your answer with algebra tiles and a drawing.

4. $3(x + 3) = 3x + 3$
5. $x(3 + 2) = 3x + 2x$
6. Rachel says that $3(x + 4) = 3x + 12$, but José says that $3(x + 4) = 3x + 4$. Use words and models to explain who is correct and why.

You can apply the Distributive Property to algebraic expressions.

---

Reading Math

The expression $5(g - 9)$ is read **5 times the quantity $g$ minus 9** or **5 times the difference of $g$ and 9**.
The Distributive Property and the properties of equality can be used to show that 
\[ 5n + 7n = (5 + 7)n \] 
Distributive Property
\[ = 12n \] 
Substitution

The expressions \(5n + 7n\) and \(12n\) are called equivalent expressions because they denote the same number. An expression is in simplest form when it is replaced by an equivalent expression having no like terms or parentheses.

Example 6  Combine Like Terms

Simplify each expression.

a. \(15x + 18x\)
\[ 15x + 18x = (15 + 18)x \] 
Distributive Property
\[ = 33x \] 
Substitution

b. \(10n + 3n^2 + 9n^2\)
\[ 10n + 3n^2 + 9n^2 = 10n + (3 + 9)n^2 \] 
Distributive Property
\[ = 10n + 12n^2 \] 
Substitution

The coefficient of a term is the numerical factor. For example, in \(17xy\), the coefficient is 17, and in \(\frac{3y^2}{4}\), the coefficient is \(\frac{3}{4}\). In the term \(m\), the coefficient is 1 since \(1 \cdot m = m\) by the Multiplicative Identity Property.

Check for Understanding

Concept Check

1. Explain why the Distributive Property is sometimes called The Distributive Property of Multiplication Over Addition.

2. OPEN ENDED Write an expression that has five terms, three of which are like terms and one term with a coefficient of 1.

3. FIND THE ERROR Courtney and Ben are simplifying \(4w^4 + w^4 + 3w^2 - 2w^2\).

   \[
   \begin{array}{l}
   \text{Courtney} \\
   4w^4 + w^4 + 3w^2 - 2w^2 \\
   = (4 + 1)w^4 + (3 - 2)w^2 \\
   = 5w^4 + 1w^2 \\
   = 5w^4 + w^2 \\
   \end{array}
   \]

   \[
   \begin{array}{l}
   \text{Ben} \\
   4w^4 + w^4 + 3w^2 - 2w^2 \\
   = (4 + 1)w^4 + (3 - 2)w^2 \\
   = 4w^4 + 1w^2 \\
   = 4w^4 + w^2 \\
   \end{array}
   \]

Who is correct? Explain your reasoning.

Guided Practice

Rewrite each expression using the Distributive Property. Then simplify.
4. \(6(12 - 2)\)
5. \(2(4 + t)\)
6. \((g - 9)5\)

Use the Distributive Property to find each product.
7. \(16(102)\)
8. \((3\frac{1}{17})(17)\)

Simplify each expression. If not possible, write simplified.
9. \(13m + m\)
10. \(3(x + 2x)\)
11. \(14a^2 + 13b^2 + 27\)
12. \(4(3g + 2)\)
COSMETOLOGY For Exercises 13 and 14, use the following information.
Ms. Curry owns a hair salon. One day, she gave 12 haircuts. She earned $19.95 for each and received an average tip of $2 for each haircut.

13. Write an expression to determine the total amount she earned.
14. How much did Ms. Curry earn?

Application

Practice and Apply

Rewrite each expression using the Distributive Property. Then simplify.

15. 8(5 + 7) 16. 7(13 + 12) 17. 12(9 - 5)
18. 13(10 - 7) 19. 3(2x + 6) 20. 8(3m + 4)
21. (4 + x)2 22. (5 + n)3 23. 28\(\frac{y}{7}\)
24. 27\(\frac{2b - 1}{3}\) 25. a(b - 6) 26. x(z + 3)
27. 2(a - 3b + 2c) 28. 4(8p + 4q - 7r)

OLYMPICS For Exercises 29 and 30, use the following information.
At the 2000 Summer Olympics in Australia, about 110,000 people attended events at Olympic Stadium each day while another 17,500 fans were at the aquatics center.

29. Write an expression you could use to determine the total number of people at Olympic Stadium and the Aquatic Center over 4 days.
30. What was the attendance for the 4-day period?

Use the Distributive Property to find each product.

31. 5 · 97 32. 8 · 990 33. 17 · 6
34. 24 · 7 35. 18\(\frac{2}{9}\) 36. 48\(\frac{1}{6}\)

COMMUNICATIONS For Exercises 37 and 38, use the following information.
A public relations consultant keeps a log of all contacts made by e-mail, telephone, and in person. In a typical week, she averages 5 hours using e-mail, 12 hours of meeting in person, and 18 hours on the telephone.

37. Write an expression that could be used to predict how many hours she will spend on these activities over the next 12 weeks.
38. How many hours should she plan for contacting people for the next 12 weeks?

INSURANCE For Exercises 39–41, use the table that shows the monthly cost of a company health plan.

<table>
<thead>
<tr>
<th>Available Insurance Plans—Monthly Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coverage</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Employee</td>
</tr>
<tr>
<td>Family (additional coverage)</td>
</tr>
</tbody>
</table>

39. Write an expression that could be used to calculate the cost of medical, dental, and vision insurance for an employee for 6 months.
40. How much does it cost an employee to get all three types of insurance for 6 months?
41. How much would an employee expect to pay for individual and family medical and dental coverage per year?
Maintain Your Skills

Lesson 1-5

The Distributive Property

Simplify each expression. If not possible, write simplified.

42. $2x + 9x$  
43. $4b + 5b$  
44. $5n^2 + 7n$

45. $3a^2 + 14a^2$  
46. $12(3c + 4)$  
47. $15(3x - 5)$

48. $6x^2 + 14x - 9x$  
49. $4y^3 + 3y^3 + y^4$  
50. $6(5a + 3b - 2b)$

51. $5(6m + 4n - 3n)$  
52. $x^2 + \frac{7}{8}x - \frac{x}{8}$  
53. $a + \frac{a}{5} + \frac{2}{5}a$

54. CRITICAL THINKING  The expression $2(\ell + w)$ may be used to find the perimeter of a rectangle. What are the length and width of a rectangle if the area is $13\frac{1}{2}$ square units and the length of one side is $\frac{1}{5}$ the measure of the perimeter?

55. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

How can the Distributive Property be used to calculate quickly?

Include the following in your answer:

• a comparison of the two methods of finding the total video game sales.

56. Simplify $3(x + y) + 2(x + y) - 4x$.

(A) $5x + y$  
(B) $9x + 5y$  
(C) $5x + 9y$  
(D) $x + 5y$

57. If $a = 2.8$ and $b = 4.2$, find the value of $c$ in the equation $c = 7(2a + 3b)$.

(A) 18.2  
(B) 238.0  
(C) 127.4  
(D) 51.8

Standardized Test Practice

代办考试

Mixed Review  Name the property illustrated by each statement or equation. (Lesson 1-4)

58. If $7 \cdot 2 = 14$, then $14 = 7 \cdot 2$.  
59. $8 + (3 + 9) = 8 + 12$  
60. $mnp = 1mnp$  
61. $3(\frac{5^2 \cdot 1}{25}) = 3 \cdot 1$

62. $\frac{3}{4} \cdot \frac{4}{3} = 1$  
63. $32 + 21 = 32 + 21$

PHYSICAL SCIENCE  For Exercises 64 and 65, use the following information.

Sound travels 1129 feet per second through air. (Lesson 1-3)

64. Write an equation that represents how many feet sound can travel in 2 seconds when it is traveling through air.

65. How far can sound travel in 2 seconds when traveling through air?

Evaluate each expression if $a = 4$, $b = 6$, and $c = 3$. (Lesson 1–2)

66. $3ab - c^2$  
67. $8(a - c)^3 + 3$  
68. $\frac{6ab}{c(a + 2)}$  
69. $(a+c)(\frac{a+b}{2})$

Getting Ready for the Next Lesson  PREREQUISITE SKILL  Find the area of each figure. (To review finding area, see pages 813 and 814.)

70.  
71.  
72.
Commutative and Associative Properties

What You’ll Learn

- Recognize the Commutative and Associative Properties.
- Use the Commutative and Associative Properties to simplify expressions.

How can properties help you determine distances?

The South Line of the Atlanta subway leaves Five Points and heads for Garnett, 0.4 mile away. From Garnett, West End is 1.5 miles. The distance from Five Points to West End can be found by evaluating the expression 0.4 + 1.5. Likewise, the distance from West End to Five Points can be found by evaluating the expression 1.5 + 0.4.

**COMMUTATIVE AND ASSOCIATIVE PROPERTIES**

In the situation above, the distance from Five Points to West End is the same as the distance from West End to Five Points. This distance can be represented by the following equation.

\[
\text{The distance from Five Points to West End} \quad \text{equals} \quad \text{the distance from West End to Five Points.}
\]

\[
0.4 + 1.5 = 1.5 + 0.4
\]

This is an example of the **Commutative Property**.

**Key Concept**

**Commutative Property**

- **Words**: The order in which you add or multiply numbers does not change their sum or product.
- **Symbols**: For any numbers \(a\) and \(b\), \(a + b = b + a\) and \(a \cdot b = b \cdot a\).
- **Examples**: \(5 + 6 = 6 + 5\), \(3 \cdot 2 = 2 \cdot 3\)

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

**Key Concept**

**Associative Property**

- **Words**: The way you group three or more numbers when adding or multiplying does not change their sum or product.
- **Symbols**: For any numbers \(a\), \(b\), and \(c\), \((a + b) + c = a + (b + c)\) and \((ab)c = a(bc)\).
- **Examples**: \((2 + 4) + 6 = 2 + (4 + 6), (3 \cdot 5) \cdot 4 = 3 \cdot (5 \cdot 4)\)
Lesson 1-6
Commutative and Associative Properties

**Example 1** Multiplication Properties

Evaluate \(8 \cdot 2 \cdot 3 \cdot 5\).

You can rearrange and group the factors to make mental calculations easier.

\[
8 \cdot 2 \cdot 3 \cdot 5 = 8 \cdot 3 \cdot 2 \cdot 5 \quad \text{Commutative (×)} \\
= (8 \cdot 3) \cdot (2 \cdot 5) \quad \text{Associative (×)} \\
= 24 \cdot 10 \quad \text{Multiply.} \\
= 240 \quad \text{Multiply.}
\]

**Example 2** Use Addition Properties

**TRANSPORTATION** Refer to the application at the beginning of the lesson. Find the distance between Five Points and Lakewood/Ft. McPherson.

<table>
<thead>
<tr>
<th>Five Points to Garnett</th>
<th>Garnett to West End</th>
<th>West End to Oakland City</th>
<th>Oakland City to Lakewood/Ft. McPherson</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.5 + 1.5 + 1.1</td>
<td>4.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

\[
0.4 + 1.5 + 1.5 + 1.1 = 0.4 + 1.1 + 1.5 + 1.5 \quad \text{Commutative (+)} \\
= (0.4 + 1.1) + (1.5 + 1.5) \quad \text{Associative (+)} \\
= 1.5 + 3.0 \quad \text{Add.} \\
= 4.5 \quad \text{Add.}
\]

Lakewood/Ft. McPherson is 4.5 miles from Five Points.

**SIMPLIFY EXPRESSIONS** The Commutative and Associative Properties can be used with other properties when evaluating and simplifying expressions.

**Concept Summary**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>(a + b = b + a)</td>
<td>(ab = ba)</td>
</tr>
<tr>
<td>Associative</td>
<td>((a + b) + c = a + (b + c))</td>
<td>((ab)c = a(bc))</td>
</tr>
<tr>
<td>Identity</td>
<td>0 is the identity.</td>
<td>1 is the identity.</td>
</tr>
<tr>
<td></td>
<td>(a + 0 = 0 + a = a)</td>
<td>(a \cdot 1 = 1 \cdot a = a)</td>
</tr>
<tr>
<td>Zero</td>
<td>(a \cdot 0 = 0 \cdot a = 0)</td>
<td></td>
</tr>
<tr>
<td>Distributive</td>
<td>(a(b + c) = ab + ac)</td>
<td>((b + c)a = ba + ca)</td>
</tr>
<tr>
<td>Substitution</td>
<td>If (a = b), then (a) may be substituted for (b).</td>
<td></td>
</tr>
</tbody>
</table>

**Example 3** Simplify an Expression

Simplify \(3c + 5(2 + c)\).

\[
3c + 5(2 + c) = 3c + 5(2) + 5(c) \quad \text{Distributive Property} \\
= 3c + 10 + 5c \quad \text{Multiply.} \\
= 3c + 5c + 10 \quad \text{Commutative (+)} \\
= (3c + 5c) + 10 \quad \text{Associative (+)} \\
= (3 + 5)c + 10 \quad \text{Distributive Property} \\
= 8c + 10 \quad \text{Substitution}
\]
**Check for Understanding**

**Concept Check**
1. Define the Associative Property in your own words.
2. Write a short explanation as to whether there is a Commutative Property of Division.
3. **OPEN ENDED** Write examples of the Commutative Property of Addition and the Associative Property of Multiplication using 1, 5, and 8 in each.

**Guided Practice**
Evaluate each expression.

4. $14 + 18 + 26$
5. $3\frac{1}{2} + 4 + 2\frac{1}{2}$
6. $5 \cdot 3 \cdot 6 \cdot 4$
7. $\frac{5}{6} \cdot 16 \cdot 9\frac{3}{4}$

Simplify each expression.

8. $4x + 5y + 6x$
9. $5a + 3b + 2a + 7b$
10. $\frac{1}{4}q + 2q + 2\frac{3}{4}q$
11. $3(4x + 2) + 2x$
12. $7(ac + 2b) + 2ac$
13. $3(x + 2y) + 4(3x + y)$

14. Write an algebraic expression for half the sum of $p$ and $2q$ increased by three-fourths $q$. Then simplify, indicating the properties used.

**Application**
15. **GEOMETRY** Find the area of the large triangle if each smaller triangle has a base measuring 5.2 centimeters and a height of 7.86 centimeters.

---

**Practice and Apply**

Evaluate each expression.

16. $17 + 6 + 13 + 24$
17. $8 + 14 + 22 + 9$
18. $4.25 + 3.50 + 8.25$
19. $6.2 + 4.2 + 4.3 + 5.8$
20. $6\frac{1}{2} + 3 + \frac{1}{2} + 2$
21. $2\frac{3}{8} + 4 + 3\frac{3}{8}$
22. $5 \cdot 11 \cdot 4 \cdot 2$
23. $3 \cdot 10 \cdot 6 \cdot 3$
24. $0.5 \cdot 2.4 \cdot 4$
25. $8 \cdot 1.6 \cdot 2.5$
26. $3\frac{3}{7} \cdot 14 \cdot 1\frac{1}{4}$
27. $2\frac{5}{8} \cdot 24 \cdot 6\frac{2}{3}$
For Exercises 28 and 29, use the following information. Hotels often have different rates for weeknights and weekends. The rates of one hotel are listed in the table.

28. If a traveler checks into the hotel on Friday and checks out the following Tuesday morning, what is the total cost of the room?

29. Suppose there is a sales tax of $5.40 for weeknights and $5.10 for weekends. What is the total cost of the room including tax?

For Exercises 30 and 31, use the following information. A video store rents new release videos for $4.49, older videos for $2.99, and DVDs for $3.99. The store also sells its used videos for $9.99.

30. Write two expressions to represent the total sales of a clerk after renting 2 DVDs, 3 new releases, 2 older videos, and selling 5 used videos.

31. What are the total sales of the clerk?

Simplify each expression.

32. \(4a + 2b + a\)

33. \(2y + 2x + 8y\)

34. \(x^2 + 3x + 2x + 5x^2\)

35. \(4a^3 + 6a + 3a^3 + 8a\)

36. \(6x + 2(2x + 7)\)

37. \(5n + 4(3n + 9)\)

38. \(3(x + 2y) + 4(3x + y)\)

39. \(3.2(x + y) + 2.3(x + y) + 4x\)

40. \(3(4m + n) + 2m\)

41. \(6(0.4f + 0.2g) + 0.5f\)

42. \(\frac{3}{4} + \frac{2}{3}(s + 2t) + s\)

43. \(2p + \frac{3}{5}(\frac{1}{2}p + 2q) + \frac{2}{3}\)

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

44. twice the sum of \(s\) and \(t\) decreased by \(s\)

45. five times the product of \(x\) and \(y\) increased by \(3xy\)

46. the product of six and the square of \(z\), increased by the sum of seven, \(z^2\), and 6

47. six times the sum of \(x\) and \(y\) squared decreased by three times the sum of \(x\) and half of \(y\) squared

48. CRITICAL THINKING Tell whether the Commutative Property always, sometimes, or never holds for subtraction. Explain your reasoning.

49. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. How can properties help you determine distances?

Include the following in your answer:

- an expression using the Commutative and Associative Properties to determine the distance from the airport to Five Points, and

- an explanation of how the Commutative and Associative Properties are useful in performing calculations.

Stop | Distance from Previous Stop
--- | ---
Five Points | 0
Garnett | 0.4
West End | 1.5
Oakland City | 1.5
Lakewood/ Ft. McPherson | 1.1
East Point | 1.9
College Park | 1.8
Airport | 0.8
50. Simplify $6(ac + 2b) + 2ac$.
   - A. $10ab + 2ac$
   - B. $12ac + 20b$
   - C. $8ac + 12b$
   - D. $12abc + 2ac$

51. Which property can be used to show that the areas of the two rectangles are equal?
   - A. Associative
   - B. Commutative
   - C. Distributive
   - D. Reflexive

52. Simplify each expression. (Lesson 1-5)
   - 52. $5(2 + x) + 7x$
   - 53. $3(5 + 2p)$
   - 54. $3(a + 2b) - 3a$
   - 55. $7m + 6(n + m)$
   - 56. $(d + 5)f + 2f$
   - 57. $t^2 + 2f^2 + 4t$

58. Name the property used in each step. (Lesson 1-4)
   - $3(10 - 5 \cdot 2) + 21 \div 7 = 3(10 - 10) + 21 \div 7$
   - $= 3(0) + 21 \div 7$
   - $= 0 + 21 \div 7$
   - $= 0 + 3$
   - $= 3$

59. Evaluate each expression. (Lesson 1-2)
   - 59. $12(5) - 6(4)$
   - 60. $7(0.2 + 0.5) - 0.6$
   - 61. $8[6^2 - 3(2 + 5)] \div 8 + 3$

62. If $x = 4$, then $2x + 7 = \underline{?}$.  
63. If $x = 8$, then $6x + 12 = \underline{?}$.  
64. If $n = 6$, then $5n - 14 = \underline{?}$.  
65. If $n = 7$, then $3n - 8 = \underline{?}$.  
66. If $a = 2$, and $b = 5$, then $4a + 3b = \underline{?}$.  

## Practice Quiz 2

Write the letters of the properties given in the right-hand column that match the examples in the left-hand column.

1. $28 + 0 = 28$
   - a. Distributive Property
2. $(18 - 7)6 = 11(6)$
   - b. Multiplicative Property of 0
3. $24 + 15 = 15 + 24$
   - c. Substitution Property of Equality
4. $8 \cdot 5 = 5 \cdot 8$
   - d. Multiplicative Identity Property
5. $(9 + 3) + 8 = 9 + (3 + 8)$
   - e. Multiplicative Inverse Property
6. $1(57) = 57$
   - f. Reflexive Property of Equality
7. $14 \cdot 0 = 0$
   - g. Associative Property
8. $3(13 + 10) = 3(13) + 3(10)$
   - h. Symmetric Property of Equality
9. If $12 + 4 = 16$, then $16 = 12 + 4$.
   - i. Commutative Property
10. $\frac{2}{3} \cdot 5 = 1$
    - j. Additive Identity Property
The statement *If the popcorn burns, then the heat was too high or the kernels heated unevenly* is called a conditional statement. Conditional statements can be written in the form *If A, then B*. Statements in this form are called *if-then statements*.

### Example 1: Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

**a. If it is Friday, then Madison and Miguel are going to the movies.**

Recall that the hypothesis is the part of the conditional following the word *if* and the conclusion is the part of the conditional following the word *then*.

- **Hypothesis**: it is Friday
- **Conclusion**: Madison and Miguel are going to the movies

**b. If $4x + 3 > 27$, then $x > 6$.**

- **Hypothesis**: $4x + 3 > 27$
- **Conclusion**: $x > 6$
Sometimes a conditional statement is written without using the words if and then. But a conditional statement can always be rewritten as an if-then statement. For example, the statement When it is not raining, I ride my bike can be written as If it is not raining, then I ride my bike.

**Example 2 Write a Conditional in If-Then Form**

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. I will go to the ball game with you on Saturday.
   - Hypothesis: it is Saturday
   - Conclusion: I will go to the ball game with you
   - If it is Saturday, then I will go to the ball game with you.

b. For a number $x$ such that $6x - 8 = 16$, $x = 4$.
   - Hypothesis: $6x - 8 = 16$
   - Conclusion: $x = 4$
   - If $6x - 8 = 16$, then $x = 4$.

**DEDUCTIVE REASONING AND COUNTEREXAMPLES**

Deductive reasoning is the process of using facts, rules, definitions, or properties to reach a valid conclusion. Suppose you have a true conditional and you know that the hypothesis is true for a given case. Deductive reasoning allows you to say that the conclusion is true for that case.

**Example 3 Deductive Reasoning**

Determine a valid conclusion that follows from the statement “If two numbers are odd, then their sum is even” for the given conditions. If a valid conclusion does not follow, write no valid conclusion and explain why.

a. The two numbers are 7 and 3.
   - 7 and 3 are odd, so the hypothesis is true.
   - Conclusion: The sum of 7 and 3 is even.
   - **CHECK** $7 + 3 = 10$ ✓ The sum, 10, is even.

b. The sum of two numbers is 14.
   - The conclusion is true. If the numbers are 11 and 3, the hypothesis is true also. However, if the numbers are 8 and 6, the hypothesis is false. There is no way to determine the two numbers. Therefore, there is no valid conclusion.

Not all if-then statements are always true or always false. Consider the statement “If Luke is listening to CDs, then he is using his portable CD player.” Luke may be using his portable CD player. However, he could also be using a computer, a car CD player, or a home CD player.

To show that a conditional is false, we can use a counterexample. A counterexample is a specific case in which a statement is false. It takes only one counterexample to show that a statement is false.
Lesson 1-7 Logical Reasoning

Example 4 Find Counterexamples

Find a counterexample for each conditional statement.

a. If you are using the Internet, then you own a computer.
   
   You could be using the Internet on a computer at a library.

b. If the Commutative Property holds for multiplication, then it holds for division.
   
   \[2 \div 1 \neq 1 \div 2\]
   
   \[2 \neq 0.5\]

Example 5 Find a Counterexample

Multiple-Choice Test Item

Which numbers are counterexamples for the statement below?

If \(x \div y = 1\), then \(x\) and \(y\) are whole numbers.

- A \(x = 2, y = 2\)
- B \(x = 0.25, y = 0.25\)
- C \(x = 1.2, y = 0.6\)
- D \(x = 6, y = 3\)

Read the Test Item

Find the values of \(x\) and \(y\) that make the statement false.

Solve the Test Item

Replace \(x\) and \(y\) in the equation \(x \div y = 1\) with the given values.

- A \(x = 2, y = 2\)
  
  \[
  \begin{align*}
  2 \div 2 & \neq 1 \\
  2 & \neq 1 \quad \checkmark
  \end{align*}
  \]

  The hypothesis is true and both values are whole numbers. The statement is true.

- B \(x = 0.25, y = 0.25\)
  
  \[
  \begin{align*}
  0.25 \div 0.25 & \neq 1 \\
  1 & \neq 1 \quad \checkmark
  \end{align*}
  \]

  The hypothesis is true, but 0.25 is not a whole number. Thus, the statement is false.

- C \(x = 1.2, y = 0.6\)
  
  \[
  \begin{align*}
  1.2 \div 0.6 & \neq 1 \\
  2 & \neq 1
  \end{align*}
  \]

  The hypothesis is false, and the conclusion is false. However, this is not a counterexample. A counterexample is a case where the hypothesis is true and the conclusion is false.

- D \(x = 6, y = 3\)
  
  \[
  \begin{align*}
  6 \div 3 & \neq 1 \\
  2 & \neq 1
  \end{align*}
  \]

  The hypothesis is false. Therefore, this is not a counterexample.

The only values that prove the statement false are \(x = 0.25\) and \(y = 0.25\). So these numbers are counterexamples. The answer is B.

Check for Understanding

Concept Check

1. OPEN ENDED Write a conditional statement and label its hypothesis and conclusion.

2. Explain why counterexamples are used.

3. Explain how deductive reasoning is used to show that a conditional is true or false.
Identify the hypothesis and conclusion of each statement.

4. If it is January, then it might snow.
5. If you play tennis, then you run fast.
6. If $34 - 3x = 16$, then $x = 6$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

7. Lance watches television when he does not have homework.
8. A number that is divisible by 10 is also divisible by 5.
9. A rectangle is a quadrilateral with four right angles.

Determine a valid conclusion that follows from the statement If the last digit of a number is 2, then the number is divisible by 2 for the given conditions. If a valid conclusion does not follow, write no valid conclusion and explain why.

10. The number is 10,452.
11. The number is divisible by 2.
12. The number is 946.

Find a counterexample for each statement.

13. If Anna is in school, then she has a science class.
14. If you can read 8 pages in 30 minutes, then you can read any book in a day.
15. If a number $x$ is squared, then $x^2 > x$.
16. If $3x + 7 \geq 52$, then $x > 15$.

17. Which number is a counterexample for the statement $x^2 > x$?

- A 1
- B 4
- C 5
- D 8

Practice and Apply

Identify the hypothesis and conclusion of each statement.

18. If both parents have red hair, then their children have red hair.
19. If you are in Hawaii, then you are in the tropics.
20. If $2n - 7 > 25$, then $n > 16$.
21. If $4(b + 9) \leq 68$, then $b \leq 8$.
22. If $a = b$, then $b = a$.
23. If $a = b$ and $b = c$, then $a = c$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

24. The trash is picked up on Monday.
25. Greg will call after school.
26. A triangle with all sides congruent is an equilateral triangle.
27. The sum of the digits of a number is a multiple of 9 when the number is divisible by 9.
28. For $x = 8$, $x^2 - 3x = 40$.
29. $4s + 6 > 42$ when $s > 9$. 
Determine whether a valid conclusion follows from the statement *If a VCR costs less than $150, then Ian will buy one* for the given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

30. A VCR costs $139.
32. Ian will not buy a VCR.
33. The price of a VCR is $199.
34. A DVD player costs $229.
35. Ian bought 2 VCRs.

Find a counterexample for each statement.

36. If you were born in Texas, then you live in Texas.
37. If you are a professional basketball player, then you play in the United States.
38. If a baby is wearing blue clothes, then the baby is a boy.
39. If a person is left-handed, then each member of that person’s family is left-handed.
40. If the product of two numbers is even, then both numbers must be even.
41. If a whole number is greater than 7, then two times the number is greater than 16.
42. If $4n - 8 \geq 52$, then $n > 15$.
43. If $x \cdot y = 1$, then $x$ or $y$ must equal 1.

**GEOMETRY** For Exercises 44 and 45, use the following information.

If points $P$, $Q$, and $R$ lie on the same line, then $Q$ is between $P$ and $R$.

44. Copy the diagram. Label the points so that the conditional is true.
45. Copy the diagram. Provide a counterexample for the conditional.

**RESEARCH** On Groundhog Day (February 2) of each year, some people say that if a groundhog comes out of its hole and sees its shadow, then there will be six more weeks of winter weather. However, if it does not see its shadow, then there will be an early spring. Use the Internet or another resource to research the weather on Groundhog Day for your city for the past 10 years. Summarize your data as examples or counterexamples for this belief.

**NUMBER THEORY** For Exercises 47–49, use the following information.

Copy the Venn diagram and place the numbers 1 to 25 in the appropriate places on the diagram.

47. What conclusions can you make about the numbers and where they appear on the diagram?
48. What conclusions can you form about numbers that are divisible by 2 and 3?
49. Find a counterexample for your conclusions, if possible.
50. **CRITICAL THINKING**  Determine whether the following statement is always true. If it is not, provide a counterexample.

*If the mathematical operation * is defined for all numbers a and b as \( a * b = a + 2b \), then the operation * is commutative.*

51. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

**How is logical reasoning helpful in cooking?**

Include the following in your answer:

- the hypothesis and conclusion of the statement *If you have small, underpopped kernels, then you have not used enough oil in your pan,* and
- examples of conditional statements used in cooking food other than popcorn.

**Standardized Test Practice**

**GRID IN**  What value of \( n \) makes the following statement true?

*If \( 14n - 12 \geq 100 \), then \( n \geq ? \).*

**53.** If \( # \) is defined as \( #x = \frac{x^3}{2} \), what is the value of \( #4 \)?

\[ A \) 8  \]
\[ B \) 16  \]
\[ C \) 32  \]
\[ D \) 64  

---

**Maintain Your Skills**

**Mixed Review**  Simplify each expression.  *(Lesson 1-6)*

54. \( 2x + 5y + 9x \)  55. \( a + 9b + 6b \)  56. \( \frac{3}{4}g + \frac{2}{5}f + \frac{5}{8}g \)

57. \( 4(5mn + 6) + 3mn \)  58. \( 2(3a + b) + 3b + 4 \)  59. \( 6x^2 + 5x + 3(2x^2) + 7x \)

**60. ENVIRONMENT**  According to the U.S. Environmental Protection Agency, a typical family of four uses 100 gallons of water flushing the toilet each day, 80 gallons of water showering and bathing, and 8 gallons of water using the bathroom sink. Write two expressions that represent the amount of water a typical family of four uses for these purposes in \( d \) days.  *(Lesson 1-5)*

**Name the property used in each expression. Then find the value of \( n \).**  *(Lesson 1-4)*

61. \( 1(n) = 64 \)  62. \( 12 + 7 = n + 12 \)  63. \( (9 - 7)5 = 2n \)

64. \( \frac{1}{4}n = 1 \)  65. \( n + 18 = 18 \)  66. \( 36n = 0 \)

**Solve each equation.**  *(Lesson 1-3)*

67. \( 5(7) + 6 = x \)  68. \( 7(4^2) - 6^2 = m \)  69. \( p = \frac{22 - (13 - 5)}{28 \div 2^2} \)

**Write an algebraic expression for each verbal expression.**  *(Lesson 1-1)*

70. the product of 8 and a number \( x \) raised to the fourth power

71. three times a number \( n \) decreased by 10

72. twelve more than the quotient of a number \( a \) and 5

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Evaluate each expression. Round to the nearest tenth. *(To review percents, see pages 802 and 803.)*

73. 40% of 90  74. 23% of 2500  75. 18% of 950

76. 38% of 345  77. 42.7% of 528  78. 67.4% of 388
**What You’ll Learn**

- Interpret graphs of functions.
- Draw graphs of functions.

**Vocabulary**

- function
- coordinate system
- x-axis
- y-axis
- origin
- ordered pair
- x-coordinate
- y-coordinate
- independent variable
- dependent variable
- relation
- domain
- range

**How**

**can real-world situations be modeled using graphs and functions?**

The graph shows the relationship between blood flow to the brain and the number of days after the concussion. The graph shows that as the number of days increases, the percent of blood flow increases.

The return of normal blood flow to the brain is said to be a function of the number of days since the concussion.

**INTERPRET GRAPHS**  A **function** is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input.

A function is graphed using a **coordinate system**. It is formed by the intersection of two number lines, the **horizontal axis** and the **vertical axis**.

Each input \( x \) and its corresponding output \( y \) can be represented on a graph using ordered pairs. An **ordered pair** is a set of numbers, or coordinates, written in the form \((x, y)\). The \( x \) value, called the **x-coordinate**, corresponds to the \( x \)-axis and the \( y \) value, or **y-coordinate**, corresponds to the \( y \)-axis.

**Example 1**  **Identify Coordinates**

**MEDICINE**  Refer to the application above. Name the ordered pair at point \( C \) and explain what it represents.

Point \( C \) is at 2 along the \( x \)-axis and about 80 along the \( y \)-axis. So, its ordered pair is \((2, 80)\). This represents 80% normal blood flow 2 days after the injury.
In Example 1, the percent of normal blood flow depends on the number of days from the injury. Therefore, the number of days from the injury is called the **independent variable** or *quantity*, and the percent of normal blood flow is called the **dependent variable** or *quantity*. Usually the independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis.

### Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each function.

a. In general, the average price of gasoline slowly and steadily increases throughout the year.

   Time is the independent variable as it is unaffected by the price of gasoline, and the price is the dependent quantity as it is affected by time.

b. The profit that a business makes generally increases as the price of their product increases.

   In this case, price is the independent quantity. Profit is the dependent quantity as it is affected by the price.

Functions can be graphed without using a scale on either axis to show the general shape of the graph that represents a function.

### Example 3 Analyze Graphs

a. The graph at the right represents the speed of a school bus traveling along its morning route. Describe what is happening in the graph.

   At the origin, the bus is stopped. It accelerates and maintains a constant speed. Then it begins to slow down, eventually stopping. After being stopped for a short time, the bus accelerates again. The starting and stopping process repeats continually.

b. Identify the graph that represents the altitude of a space shuttle above Earth, from the moment it is launched until the moment it lands.

   Before it takes off, the space shuttle is on the ground. It blasts off, gaining altitude until it reaches space where it orbits Earth at a constant height until it comes back to Earth. Graph A shows this situation.
**DRAW GRAPHS**  
Graphs can be used to represent many real-world situations.

**Example 4** Draw Graphs

An electronics store is having a special sale. For every two DVDs you buy at the regular price of $29 each, you get a third DVD free.

a. Make a table showing the cost of buying 1 to 5 DVDs.

<table>
<thead>
<tr>
<th>Number of DVDs</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>87</td>
</tr>
<tr>
<td>5</td>
<td>116</td>
</tr>
</tbody>
</table>

b. Write the data as a set of ordered pairs.

The ordered pairs can be determined from the table. The number of DVDs is the independent variable, and the total cost is the dependent variable. So, the ordered pairs are (1, 29), (2, 58), (3, 58), (4, 87), and (5, 116).

c. Draw a graph that shows the relationship between the number of DVDs and the total cost.

A set of ordered pairs, like those in Example 4, is called a **relation**. The set of the first numbers of the ordered pairs is the **domain**. The domain contains all values of the independent variable. The set of second numbers of the ordered pairs is the **range** of the relation. The range contains all values of the dependent variable.

**Example 5** Domain and Range

**JOBS**  
Rasha earns $6.75 per hour working up to 4 hours each day after school. Her weekly earnings are a function of the number of hours she works.

a. Identify a reasonable domain and range for this situation.

The domain contains the number of hours Rasha works each week. Since she works up to 4 hours each weekday, she works up to $5 \times 4$ or 20 hours a week. Therefore, a reasonable domain would be values from 0 to 20 hours. The range contains her weekly earnings from $0$ to $20 \times $6.75 or $135. Thus, a reasonable range is $0$ to $135$.

b. Draw a graph that shows the relationship between the number of hours Rasha works and the amount she earns each week.

Graph the ordered pairs (0, 0) and (20, 135). Since she can work any amount of time up to 20 hours, connect the two points with a line to include those points.
Check for Understanding

Concept Check
1. Explain why the order of the numbers in an ordered pair is important.
2. Describe the difference between dependent and independent variables.
3. OPEN ENDED Give an example of a relation. Identify the domain and range.

Guided Practice
4. The graph at the right represents Alexi’s speed as he rides his bike. Give a description of what is happening in the graph.

5. Identify the graph that represents the height of a skydiver just before she jumps from a plane until she lands.

Applications PHYSICAL SCIENCE For Exercises 6–8, use the table and the information.
During an experiment, the students of Ms. Roswell’s class recorded the height of an object above the ground at several intervals after it was dropped from a height of 5 meters. Their results are in the table below.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>500</td>
<td>480</td>
<td>422</td>
<td>324</td>
<td>186</td>
<td>10</td>
</tr>
</tbody>
</table>

6. Identify the independent and dependent variables.
7. Write a set of ordered pairs representing the data in the table.
8. Draw a graph showing the relationship between the height of the falling object and time.

9. BASEBALL Paul is a pitcher for his school baseball team. Draw a reasonable graph that shows the height of the baseball from the ground from the time he releases the ball until the time the catcher catches the ball. Let the horizontal axis show the time and the vertical axis show the height of the ball.

Practice and Apply

10. The graph below represents Michelle’s temperature when she was sick. Describe what is happening in the graph.

11. The graph below represents the balance in Rashaad’s checking account. Describe what is happening in the graph.
12. **TOYS** Identify the graph that displays the speed of a radio-controlled car as it moves along and then hits a wall.

![Graphs A, B, C](image)

13. **INCOME** In general, as a person gets older, their income increases until they retire. Which of the graphs below represents this?

![Graphs A, B, C](image)

14. Write the ordered pairs with whole-number coordinates that represent the cost of parking for up to 36 hours.

15. Draw a graph to show the cost of parking for up to 36 hours.

16. What is the cost of parking if you arrive on Monday at 7:00 A.M. and depart on Tuesday at 9:00 P.M.?

17. Identify the independent and dependent variables.

18. Draw a graph of the data.

19. Use the data to predict the sum of the measures of the interior angles for an octagon, nonagon, and decagon.

20. **CARS** A car was purchased new in 1970. The owner has taken excellent care of the car, and it has relatively low mileage. Draw a reasonable graph to show the value of the car from the time it was purchased to the present.

21. **CHEMISTRY** When ice is exposed to temperatures above 32°F, it begins to melt. Draw a reasonable graph showing the relationship between the temperature of a block of ice as it is removed from a freezer and placed on a counter at room temperature. (*Hint:* The temperature of the water will not exceed the temperature of its surroundings.)
22. **CRITICAL THINKING** Mallory is 23 years older than Lisa.
   a. Draw a graph showing Mallory’s age as a function of Lisa’s age for the first 40 years of Lisa’s life.
   b. Find the point on the graph when Mallory is twice as old as Lisa.

23. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

   **How can real-world situations be modeled using graphs and functions?**
   Include the following in your answer:
   • an explanation of how the graph helps you analyze the situation,
   • a summary of what happens during the first 24 hours from the time of a concussion, and
   • an explanation of the time in which significant improvement occurs.

24. The graph shows the height of a model rocket shot straight up. How many seconds did it take for the rocket to reach its maximum height?
   - **A** 3 
   - **B** 4 
   - **C** 5 
   - **D** 6

25. Andre owns a computer backup service. He charges his customers $2.50 for each backup CD. His expenses include $875 for the CD recording equipment and $0.35 for each blank CD. Which equation could Andre use to calculate his profit $p$ for the recording of $n$ CDs?
   - **A** $p = 2.15n - 875$
   - **B** $p = 2.85 + 875$
   - **C** $p = 2.50 - 875.65$
   - **D** $p = 875 - 2.15n$

**Maintain Your Skills**

**Mixed Review** Identify the hypothesis and conclusion of each statement. *(Lesson 1-7)*
26. You can send e-mail with a computer.
27. The express lane is for shoppers who have 9 or fewer items.
28. Name the property used in each step. *(Lesson 1-6)*
   
   \[
   ab(a + b) = (ab)a + (ab)b \\
   = a(ab) + (ab)b \\
   = (a \cdot a)b + a(b \cdot b) \\
   = a^2b + ab^2
   \]

Name the property used in each statement. Then find the value of $n$. *(Lesson 1-4)*
29. $(12 - 9)(4) = n(4)$
30. $7(n) = 0$
31. $n(87) = 87$

**Getting Ready for the Next Lesson**
32. **PREREQUISITE SKILL** Use the information in the table to construct a bar graph. *(To review making bar graphs, see pages 806 and 807.)*

<table>
<thead>
<tr>
<th>Format</th>
<th>country</th>
<th>adult</th>
<th>contemporary</th>
<th>news/talk</th>
<th>oldies</th>
<th>rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>2249</td>
<td>1557</td>
<td>1426</td>
<td>1135</td>
<td>827</td>
<td></td>
</tr>
</tbody>
</table>

**Source:** The World Almanac
Investigating Real-World Functions

The table shows the number of students enrolled in elementary and secondary schools in the United States for the given years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment (thousands)</th>
<th>Year</th>
<th>Enrollment (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900</td>
<td>15,503</td>
<td>1970</td>
<td>45,550</td>
</tr>
<tr>
<td>1920</td>
<td>21,578</td>
<td>1980</td>
<td>41,651</td>
</tr>
<tr>
<td>1940</td>
<td>25,434</td>
<td>1990</td>
<td>40,543</td>
</tr>
<tr>
<td>1960</td>
<td>36,807</td>
<td>1998</td>
<td>46,327</td>
</tr>
</tbody>
</table>

Source: The World Almanac

Step 1 On grid paper, draw a vertical and horizontal axis as shown. Make your graph large enough to fill most of the sheet. Label the horizontal axis 0 to 120 and the vertical axis 0 to 60,000.

Step 2 To make graphing easier, let $x$ represent the number of years since 1900. Write the eight ordered pairs using this method. The first will be $(0, 15,503)$.

Step 3 Graph the ordered pairs on your grid paper.

Analyze

1. Use your graph to estimate the number of students in elementary and secondary school in 1910 and in 1975.
2. Use your graph to estimate the number of students in elementary and secondary school in 2020.

Make a Conjecture

3. Describe the methods you used to make your estimates for Exercises 1 and 2.
4. Do you think your prediction for 2020 will be accurate? Explain your reasoning.
5. Graph this set of data, which shows the number of students per computer in U.S. schools. Predict the number of students per computer in 2010. Explain how you made your prediction.

<table>
<thead>
<tr>
<th>Year</th>
<th>Students per Computer</th>
<th>Year</th>
<th>Students per Computer</th>
<th>Year</th>
<th>Students per Computer</th>
<th>Year</th>
<th>Students per Computer</th>
</tr>
</thead>
</table>

Source: The World Almanac
**What You'll Learn**

- Analyze data given in tables and graphs (bar, line, and circle).
- Determine whether graphs are misleading.

**Why are graphs and tables used to display data?**

For several weeks after Election Day in 2000, data regarding the presidential vote counts changed on a daily basis.

The bar graph at the right illustrates just how close the election was at one point. The graph allows you to compare the data visually.

**ANALYZE DATA** A bar graph compares different categories of numerical information, or data, by showing each category as a bar whose length is related to the frequency. Bar graphs can also be used to display multiple sets of data in different categories at the same time. Graphs with multiple sets of data always have a key to denote which bars represent each set of data.

**Example 1** Analyze a Bar Graph

The table shows the number of men and women participating in NCAA championship sports programs from 1995 to 1999.

<table>
<thead>
<tr>
<th>Year</th>
<th>'97–'98</th>
<th>'98–'99</th>
<th>'99–'00</th>
<th>'00–'01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>200,031</td>
<td>207,592</td>
<td>208,481</td>
<td>206,573</td>
</tr>
<tr>
<td>Women</td>
<td>133,376</td>
<td>145,832</td>
<td>146,617</td>
<td>149,115</td>
</tr>
</tbody>
</table>

Source: NCAA

These same data are displayed in a bar graph.

a. Describe the general trend shown in the graph.

The graph shows that the number of men has remained fairly constant while the number of women has been increasing.
b. Approximately how many more men than women participated in sports during the 1997–1998 school year?

The bar for the number of men shows about 200,000 and the bar for the women shows about 130,000. So, there were approximately 200,000–130,000 or 70,000 more men than women participating in the 1997–1998 school year.

c. What was the total participation among men and women in the 2000–2001 academic year?

Since the table shows the exact numbers, use the data in it.

<table>
<thead>
<tr>
<th>Number of men</th>
<th>plus</th>
<th>Number of women</th>
<th>equals</th>
<th>Total participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>206,573</td>
<td></td>
<td>149,115</td>
<td></td>
<td>355,688</td>
</tr>
</tbody>
</table>

There was a total of 355,688 men and women participating in sports in the 2000–2001 academic year.

Another type of graph used to display data is a circle graph. A circle graph compares parts of a set of data as a percent of the whole set. The percents in a circle graph should always have a sum of 100%.

**Example 2 Analyze a Circle Graph**

A recent survey asked drivers in several cities across the United States if traffic in their area had gotten better, worse, or had not changed in the past five years. The results of the survey are displayed in the circle graph.

a. If 4500 people were surveyed, how many felt that traffic had improved in their area?

The section of the graph representing people who said traffic is better is 8% of the circle, so find 8% of 4500.

\[
0.08 \times 4500 = 360
\]

360 people said that traffic was better.

b. If a city with a population of 647,000 is representative of those surveyed, how many people could be expected to think that traffic conditions are worse?

63% of those surveyed said that traffic is worse, so find 63% of 647,000.

\[
0.63 \times 647,000 = 407,610
\]

Thus, 407,610 people in the city could be expected to say that traffic conditions are worse.

A third type of graph used to display data is a line graph. Line graphs are useful when showing how a set of data changes over time. They can also be helpful when making predictions.
Example 3 Analyze a Line Graph

EDUCATION Refer to the line graph below.

a. Estimate the change in enrollment between 1995 and 1999.

The enrollment for 1995 is about 14.25 million, and the enrollment for 1999 is about 14.9 million. So, the change in enrollment is 14.9 – 14.25 or 0.65 million.

b. If the rate of growth between 1998 and 1999 continues, predict the number of people who will be enrolled in higher education in the year 2005.

Based on the graph, the increase in enrollment from 1998 to 1999 is 0.3 million. So, the enrollment should increase by 0.3 million per year.

\[
14.9 + 0.3(6) = 14.9 + 1.8 \quad \text{Multiply the annual increase, 0.3, by the number of years, 6.}
\]

\[
= 16.7 \quad \text{Enrollment in 2005 should be about 16.7 million.}
\]

Concept Summary

<table>
<thead>
<tr>
<th>Type of Graph</th>
<th>bar graph</th>
<th>circle graph</th>
<th>line graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>When to Use</td>
<td>to compare different categories of data</td>
<td>to show data as parts of a whole set of data</td>
<td>to show the change in data over time</td>
</tr>
</tbody>
</table>

MISLEADING GRAPHS Graphs are very useful for displaying data. However, graphs that have been constructed incorrectly can be confusing and can lead to false assumptions. Many times these types of graphs are mislabeled, incorrect data is compared, or the graphs are constructed to make one set of data appear greater than another set. Here are some common ways that a graph may be misleading.

- Numbers are omitted on an axis, but no break is shown.
- The tick marks on an axis are not the same distance apart or do not have the same-sized intervals.
- The percents on a circle graph do not have a sum of 100.

Example 4 Misleading Graphs

AUTOMOBILES The graph shows the number of sport-utility vehicle (SUV) sales in the United States from 1990 to 1999. Explain how the graph misrepresents the data.

The vertical axis scale begins at 1 million. This causes the appearance of no vehicles sold in 1990 and 1991, and very few vehicles sold through 1994.

Source: The World Almanac
Concept Check

1. Explain the appropriate use of each type of graph.
   - circle graph
   - bar graph
   - line graph

2. OPEN ENDED Find a real-world example of a graph in a newspaper or magazine. Write a description of what the graph displays.

3. Describe ways in which a circle graph could be drawn so that it is misleading.

Guided Practice

SPORTS For Exercises 4 and 5, use the following information. There are 321 NCAA Division I schools. The graph at the right shows the sports that are offered at the most Division I schools.

4. How many more schools participate in basketball than in golf?

5. What sport is offered at the fewest schools?

EDUCATION For Exercises 6–9, use the table that shows the number of foreign students as a percent of the total college enrollment in the United States.

<table>
<thead>
<tr>
<th>Country of Origin</th>
<th>Total Student Enrollment (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.02</td>
</tr>
<tr>
<td>Canada</td>
<td>0.15</td>
</tr>
<tr>
<td>France</td>
<td>0.04</td>
</tr>
<tr>
<td>Germany</td>
<td>0.06</td>
</tr>
<tr>
<td>Italy</td>
<td>0.22</td>
</tr>
<tr>
<td>Spain</td>
<td>0.03</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Source: Statistical Abstract of the United States

6. There were about 14.9 million students enrolled in colleges in 1999. How many of these students were from Germany?

7. How many more students were from Canada than from the United Kingdom in 1999?

8. Would it be appropriate to display this data in a circle graph? Explain.

9. Would a bar or a line graph be more appropriate to display these data? Explain.
**HOME ENTERTAINMENT**  For Exercises 10 and 11, refer to the graph.

10. Describe why the graph is misleading.

11. What should be done so that the graph displays the data more accurately?

**Practise and Apply**

**VIDEOGRAPHY**  For Exercises 12 and 13, use the table that shows the average cost of preparing one hour of 35-millimeter film versus one hour of digital video.

<table>
<thead>
<tr>
<th></th>
<th>Cost in USD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Film stock</strong></td>
<td>$3110.40</td>
</tr>
<tr>
<td><strong>Processing</strong></td>
<td>$621.00</td>
</tr>
<tr>
<td><strong>Prep for telecine</strong></td>
<td>$60.00</td>
</tr>
<tr>
<td><strong>Telecine</strong></td>
<td>$1000.00</td>
</tr>
<tr>
<td><strong>Tape stock</strong></td>
<td>$73.20</td>
</tr>
</tbody>
</table>

12. What is the total cost of using 35-millimeter film?

13. Estimate how many times as great the cost of using 35-millimeter film is as using digital video.

**BOOKS**  For Exercises 14 and 15, use the graph that shows the time of year people prefer to buy books.

<table>
<thead>
<tr>
<th>Season</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>21%</td>
</tr>
<tr>
<td>Spring</td>
<td>19%</td>
</tr>
<tr>
<td>Summer</td>
<td>15%</td>
</tr>
<tr>
<td>Fall</td>
<td>44%</td>
</tr>
</tbody>
</table>

14. Suppose the total number of books purchased for the year was 25 million. Estimate the number of books purchased in the spring.

15. Suppose the manager of a bookstore has determined that she sells about 15,000 books a year. Approximately how many books should she expect to sell during the summer?

16. **ENTERTAINMENT**  The line graph shows the number of cable television systems in the United States from 1995 to 2000. Explain how the graph misrepresents the data.

![Graph showing Cable Television Systems, 1995–2000](image)
17. **FOOD**  Oatmeal can be found in 80% of the homes in the United States. The circle graph shows favorite oatmeal toppings. Is the graph misleading? If so, explain why and tell how the graph can be fixed so that it is not misleading.

18. **CRITICAL THINKING**  The table shows the percent of United States households owning a color television for the years 1980 to 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>83</td>
</tr>
<tr>
<td>1985</td>
<td>91</td>
</tr>
<tr>
<td>1990</td>
<td>98</td>
</tr>
<tr>
<td>1995</td>
<td>99</td>
</tr>
<tr>
<td>2000</td>
<td>99</td>
</tr>
</tbody>
</table>

a. Display the data in a line graph that shows little increase in ownership.

b. Draw a line graph that shows a rapid increase in the number of households owning a color television.

c. Are either of your graphs misleading? Explain.

19. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

Why are graphs and tables used to display data?

Include the following in your answer:

- a description of how to use graphs to make predictions, and
- an explanation of how to analyze a graph to determine whether the graph is misleading.

20. According to the graph, the greatest increase in temperature occurred between which two days?

   - A. 1 and 2
   - B. 6 and 7
   - C. 2 and 3
   - D. 5 and 6

21. A graph that is primarily used to show the change in data over time is called a

   - A. circle graph
   - B. bar graph
   - C. line graph
   - D. data graph

22. **PHYSICAL FITNESS**  Pedro likes to exercise regularly. On Mondays, he walks two miles, runs three miles, sprints one-half of a mile, and then walks for another mile. Sketch a graph that represents Mitchell’s heart rate during his Monday workouts.  *(Lesson 1-8)*

23. Find a counterexample for each statement.  *(Lesson 1-7)*

   - If \( x \leq 12 \), then \( 4x - 5 \leq 42 \).
   - If \( x > 1 \), then \( x < \frac{1}{x} \).
   - If the perimeter of a rectangle is 16 inches, then each side is 4 inches long.

24. Simplify each expression.  *(Lesson 1-6)*

   - \( 7a + 5b + 3b + 3a \)
   - \( 4x^2 + 9x + 2x^2 + x \)
   - \( \frac{1}{2}n + \frac{2}{3}m + \frac{1}{2}m + \frac{1}{3}n \)
Statistical Graphs

You can use a computer spreadsheet program to display data in different ways. The data is entered into a table and then displayed in your chosen type of graph.

Example

Use a spreadsheet to make a line graph of the data on sports equipment sales.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (million $)</td>
<td>150</td>
<td>268</td>
<td>377</td>
<td>545</td>
<td>646</td>
<td>590</td>
<td>562</td>
<td>515</td>
</tr>
</tbody>
</table>

Example: Statistical Graphs

Source: National Sporting Goods Association

Step 1 Enter the data in a spreadsheet. Use Column A for the years and Column B for the sales.

Step 2 Select the data to be included in your graph. Then use the graph tool to create the graph. The spreadsheet will allow you to change the appearance of the graph by adding titles and axis labels, adjusting the scales on the axes, changing colors, and so on.

Exercises

For Exercises 1–3, use the data on snowmobile sales in the table below.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (million $)</td>
<td>322</td>
<td>391</td>
<td>515</td>
<td>715</td>
<td>910</td>
<td>974</td>
<td>975</td>
<td>957</td>
</tr>
</tbody>
</table>

Source: National Sporting Goods Association

1. Use a spreadsheet program to create a line graph of the data.
2. Use a spreadsheet program to create a bar graph of the data.
3. Adjust the scales on each of the graphs that you created. Is it possible to create a misleading graph using a spreadsheet program? Explain.
Choose the letter of the property that best matches each statement.

1. For any number $a$, $a + 0 = 0 + a = a$.
   - a. Additive Identity Property

2. For any number $a$, $a \cdot 1 = 1 \cdot a = a$.
   - b. Distributive Property

3. For any number, $a \cdot 0 = 0 \cdot a = 0$.
   - c. Commutative Property

4. For any nonzero number $a$, there is exactly one number $\frac{1}{a}$ such that $\frac{1}{a} \cdot a = a \cdot \frac{1}{a} = 1$.
   - d. Associative Property

5. For any number $a$, $a = a$.
   - e. Multiplicative Identity Property

6. For any numbers $a$ and $b$, if $a = b$, then $b = a$.
   - f. Multiplicative Inverse Property

7. For any numbers $a$ and $b$, if $a = b$, then $a$ may be replaced by $b$ in any expression.
   - g. Multiplicative Property of Zero

8. For any numbers $a$, $b$, and $c$, if $a = b$ and $b = c$, then $a = c$.
   - h. Reflexive Property

9. For any numbers $a$, $b$, and $c$, $a(b + c) = ab + ac$.
   - i. Substitution Property

10. For any numbers $a$, $b$, and $c$, $a + (b + c) = (a + b) + c$.
    - j. Symmetric Property

   - k. Transitive Property
Chapter 1  Study Guide and Review

Chapter 1  Study Guide and Review

1-2
See pages 11–15.

Order of Operations

Concept Summary

Expressions must be simplified using the order of operations.

Step 1  Evaluate expressions inside grouping symbols.

Step 2  Evaluate all powers.

Step 3  Do all multiplications and/or divisions from left to right.

Step 4  Do all additions and/or subtractions from left to right.

Example
Evaluate $x^2 - (y + 2)$ if $x = 4$ and $y = 3$.

$x^2 - (y + 2) = 4^2 - (3 + 2) = 4^2 - 5 = 16 - 5 = 11$

Evaluate power.
Subtract 5 from 16.

Exercises
Evaluate each expression.  See Examples 1–3 on pages 11 and 12.

21.  $3 + 2 \cdot 4$
22.  $\frac{(10 - 6)}{8}$
23.  $18 - 4^2 + 7$
24.  $8(2 + 5) - 6$
25.  $4(11 + 7) - 9 \cdot 8$
26.  $288 \div [3(9 + 3)]$
27.  $16 \div 2 \cdot 5 \cdot 3 + 6$
28.  $6(4^3 + 2^2)$
29.  $(3 \cdot 1)^3 - \frac{(4 + 6)}{(5 \cdot 2)}$

Evaluate each expression if $x = 3$, $t = 4$, and $y = 2$.  See Example 4 on page 12.

30.  $t^2 + 3y$
31.  $xty^3$
32.  $\frac{ty}{x}$
33.  $x + t^2 + y^2$
34.  $3ty - x^2$
35.  $8(x - y)^2 + 2t$
### 1-3 Open Sentences

**Concept Summary**
- Open sentences are solved by replacing the variables in an equation with numerical values.
- Inequalities like $x + 2 \geq 7$ are solved the same way that equations are solved.

**Example**

Solve $5^2 - 3 = y$.

$5^2 - 3 = y$  
Original equation  

$25 - 3 = y$  
Evaluate the power.  

$22 = y$  
Subtract 3 from 25.

The solution is 22.

**Exercises**  
Solve each equation.  
**See Example 2 on page 17.**

36. $x = 22 - 13$  
37. $y = 4 + 3^2$  
38. $m = \frac{64 + 4}{17}$

39. $x = \frac{21 - 3}{12 - 3}$  
40. $a = \frac{14 + 28}{4 + 3}$  
41. $n = \frac{96 \div 6}{8 \div 2}$

42. $b = \frac{7(4 \cdot 3)}{18 \div 3}$  
43. $\frac{6(7) - 2(3)}{4^2 - 6(2)}$  
44. $y = 5[2(4) - 1^3]$

Find the solution set for each inequality if the replacement set is {4, 5, 6, 7, 8}.  
**See Example 3 on page 17.**

45. $x + 2 > 7$  
46. $10 - x < 7$  
47. $2x + 5 \geq 15$

### 1-4 Identity and Equality Properties

**Concept Summary**
- Adding zero to a quantity or multiplying a quantity by one does not change the quantity.
- Using the Reflexive, Symmetric, Transitive, and Substitution Properties along with the order of operations helps in simplifying expressions.

**Example**

Evaluate $36 + 7 \cdot 1 + 5(2 - 2)$. Name the property used in each step.

$36 + 7 \cdot 1 + 5(2 - 2) = 36 + 7 \cdot 1 + 5(0)$  
Substitution  

$= 36 + 7 + 5(0)$  
Multiplicative Identity  

$= 36 + 7$  
Multiplicative Prop. of Zero  

$= 43$  
Substitution

**Exercises**  
Evaluate each expression. Name the property used in each step.  
**See Example 2 on page 23.**

48. $2[3 \div (19 - 4^2)]$  
49. $\frac{1}{2} \cdot 2 + 2[3 \cdot 3 - 1]$  
50. $4^2 - 2^2 - (4 - 2)$

51. $1.2 - 0.05 + 2^3$  
52. $(7 - 2)(5) - 5^2$  
53. $3(4 + 4)^2 - \frac{1}{4}(8)$
### 1-5 The Distributive Property

**Concept Summary**
- For any numbers \( a, b, \) and \( c, \) \( a(b + c) = ab + ac \) and \( (b + c)a = ba + ca. \)
- For any numbers \( a, b, \) and \( c, \) \( a(b - c) = ab - ac \) and \( (b - c)a = ba - ca. \)

**Examples**

1. **Rewrite** \( 5(t + 3) \) using the Distributive Property. Then simplify.
   
   \[
   5(t + 3) = 5t + 15 \quad \text{Distributive Property}
   \]

2. **Simplify** \( 2x^2 + 4x^2 + 7x. \)
   
   \[
   2x^2 + 4x^2 + 7x = (2 + 4)x^2 + 7x \quad \text{Distributive Property}
   \]
   
   \[
   = 6x^2 + 7x \quad \text{Substitution}
   \]

**Exercises**

Rewrite each product using the Distributive Property. Then simplify.

See Examples 1 and 2 on page 27.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.</td>
<td>( 2(4 + 7) )</td>
</tr>
<tr>
<td>55.</td>
<td>( 8(15 - 6) )</td>
</tr>
<tr>
<td>56.</td>
<td>( 4(x + 1) )</td>
</tr>
<tr>
<td>57.</td>
<td>( 3\left(\frac{1}{3} - p\right) )</td>
</tr>
<tr>
<td>58.</td>
<td>( 6(a + b) )</td>
</tr>
<tr>
<td>59.</td>
<td>( 8(3x - 7y) )</td>
</tr>
</tbody>
</table>

Simplify each expression. If not possible, write simplified.

See Example 6 on page 29.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.</td>
<td>( 4a + 9a )</td>
</tr>
<tr>
<td>61.</td>
<td>( 4np + 7mp )</td>
</tr>
<tr>
<td>62.</td>
<td>( 3w - w + 4v - 3v )</td>
</tr>
<tr>
<td>63.</td>
<td>( 3m + 5m + 12n - 4n )</td>
</tr>
<tr>
<td>64.</td>
<td>( 2p(1 + 16r) )</td>
</tr>
<tr>
<td>65.</td>
<td>( 9y + 3y - 5x )</td>
</tr>
</tbody>
</table>

### 1-6 Commutative and Associative Properties

**Concept Summary**
- For any numbers \( a \) and \( b, \) \( a + b = b + a \) and \( a \cdot b = b \cdot a. \)
- For any numbers \( a, b \) and \( c, \) \( (a + b) + c = a + (b + c) \) and \( (ab)c = a(bc). \)

**Example**

Simplify \( 3x + 7xy + 9x. \)

\[
3x + 7xy + 9x = 3x + 9x + 7xy \quad \text{Commutative (+)}
\]

\[
= (3 + 9)x + 7xy \quad \text{Distributive Property}
\]

\[
= 12x + 7xy \quad \text{Substitution}
\]

**Exercises**

Simplify each expression.

See Example 3 on page 33.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.</td>
<td>( 3x + 4y + 2x )</td>
</tr>
<tr>
<td>67.</td>
<td>( 7w^2 + w + 2w^2 )</td>
</tr>
<tr>
<td>68.</td>
<td>( \frac{3}{2}m + \frac{1}{2}m + n )</td>
</tr>
<tr>
<td>69.</td>
<td>( 6a + 5b + 2c + 8b )</td>
</tr>
<tr>
<td>70.</td>
<td>( 3(2 + 3x) + 21x )</td>
</tr>
<tr>
<td>71.</td>
<td>( 6(2n - 4) + 5n )</td>
</tr>
</tbody>
</table>

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

See Example 4 on page 34.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>72.</td>
<td>five times the sum of ( x ) and ( y ) decreased by 2x</td>
</tr>
<tr>
<td>73.</td>
<td>twice the product of ( p ) and ( q ) increased by the product of ( p ) and ( q )</td>
</tr>
<tr>
<td>74.</td>
<td>six times ( a ) plus the sum of eight times ( b ) and twice ( a )</td>
</tr>
<tr>
<td>75.</td>
<td>three times the square of ( x ) plus the sum of ( x ) squared and seven times ( x )</td>
</tr>
</tbody>
</table>
Logical Reasoning

Concept Summary

- Conditional statements can be written in the form If $A$, then $B$, where $A$ is
  the hypothesis and $B$ is the conclusion.
- One counterexample can be used to show that a statement is false.

Example

Identify the hypothesis and conclusion of the statement *The trumpet player must audition to be in the band.* Then write the statement in if-then form.

Hypothesis: a person is a trumpet player

Conclusion: the person must audition to be in the band

If a person is a trumpet player, then the person must audition to be in the band.

Exercises

Identify the hypothesis and conclusion of each statement. Then, write each statement in if-then form. *See Example 2 on page 38.*

- 76. School begins at 7:30 A.M.
- 77. Triangles have three sides.

Find a counterexample for each statement. *See Example 4 on page 39.*

- 78. If $x > y$, then $2x > 3y$.
- 79. If $a > b$ and $a > c$, then $b > c$.

Graphs and Functions

Concept Summary

- Graphs can be used to represent a function and to visualize data.

Example

A computer printer can print 12 pages of text per minute.

a. Make a table showing the number of pages printed in 1 to 5 minutes.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pages</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

b. Sketch a graph that shows the relationship between time and the number of pages printed.

Exercises

80. Identify the graph that represents the altitude of an airplane taking off, flying for a while, then landing. *See Example 3 on page 44.*

Graphs A, B, and C are shown.
Statistics: Analyzing Data by Using Tables and Graphs

Concept Summary

- Bar graphs are used to compare different categories of data.
- Circle graphs are used to show data as parts of a whole set of data.
- Line graphs are used to show the change in data over time.

Example

The bar graph shows ways people communicate with their friends.

a. About what percent of those surveyed chose e-mail as their favorite way to talk to friends?

The bar for e-mail is about halfway between 30% and 40%. Thus, about 35% favor e-mail.

b. What is the difference in the percent of people favoring letters and those favoring the telephone?

The bar for those favoring the telephone is at 60%, and the bar for letters is about 20%. So, the difference is 60 – 20 or 40%.

Exercises

CLASS TRIP For Exercises 84 and 85, use the circle graph and the following information.

A survey of the ninth grade class asked members to indicate their choice of locations for their class trip. The results of the survey are displayed in the circle graph. See Example 2 on page 51.

84. If 120 students were surveyed, how many chose the amusement park?

85. If 180 students were surveyed, how many more chose the amusement park than the water park?
Vocabulary and Concepts

Choose the letter of the property that best matches each statement.

1. For any number \(a\), \(a = a\).  
   a. Substitution Property of Equality  
2. For any numbers \(a\) and \(b\), if \(a = b\), then \(b\) may be replaced by \(a\) in any expression or equation.  
   b. Symmetric Property of Equality  
3. For any numbers \(a\), \(b\), and \(c\), if \(a = b\) and \(b = c\), then \(a = c\).  
   c. Transitive Property of Equality  
   d. Reflexive Property of Equality

Skills and Applications

Write an algebraic expression for each verbal expression.

4. the sum of a number \(x\) and 13  
5. the difference of 7 and a number \(x\) squared

Simplify each expression.

6. \(5(9 + 3) - 3 \cdot 4\)  
7. \(12 \cdot 6 \div 3 \cdot 2 \div 8\)

Evaluate each expression if \(a = 2\), \(b = 5\), \(c = 3\), and \(d = 1\).

8. \(a^2b + c\)  
9. \((cd)^3\)  
10. \((a + d)c\)

Solve each equation.

11. \(y = (4.5 + 0.8) - 3.2\)  
12. \(4^2 - 3(4 - 2) = x\)  
13. \(\frac{2^3 - 1^3}{2 + 1} = n\)

Evaluate each expression. Name the property used in each step.

14. \(3^2 - 2 + (2 - 2)\)  
15. \((2 \cdot 2 - 3) + 2^2 + 3^2\)

Rewrite each expression in simplest form.

16. \(2m + 3m\)  
17. \(4x + 2y - 2x + y\)  
18. \(3(2a + b) - 5a + 4b\)

Find a counterexample for each conditional statement.

19. If you run fifteen minutes today, then you will be able to run a marathon tomorrow.  
20. If \(2x - 3 < 9\), then \(x \leq 6\).

Sketch a reasonable graph for each situation.

21. A basketball is shot from the free throw line and falls through the net.  
22. A nickel is dropped on a stack of pennies and bounces off.

ICE CREAM  For Exercises 23 and 24, use the following information.

A school survey at West High School determined the favorite flavors of ice cream are chocolate, vanilla, butter pecan, and bubble gum. The results of the survey are displayed in the circle graph.

23. If 200 students were surveyed, how many more chose chocolate than vanilla?  
24. What was the total percent of students who chose either chocolate or vanilla?

25. STANDARDIZED TEST PRACTICE  Which number is a counterexample for the statement below?  
   If \(a\) is a prime number, then \(a\) is odd.
   a. 5  
   b. 4  
   c. 3  
   d. 2
Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The Maple Grove Warehouse measures 800 feet by 200 feet. If \( \frac{3}{4} \) of the floor space is covered, how many square feet are not covered?  (Prerequisite Skill)
   
   A) 4000  B) 40,000  C) 120,000  D) 160,000

2. The radius of a circular flower garden is 4 meters. How many meters of edging will be needed to surround the garden? (Prerequisite Skill)
   
   A) 7.14 m  B) 12.56 m  C) 25.12 m  D) 20.24 m

3. The Johnson family spends about $80 per week on groceries. Approximately how much do they spend on groceries per year? (Prerequisite Skill)
   
   A) $400  B) $4000  C) $8000  D) $40,000

4. Daria is making 12 party favors for her sister’s birthday party. She has 50 stickers, and she wants to use as many of them as possible. If she puts the same number of stickers in each bag, how many stickers will she have left over? (Prerequisite Skill)
   
   A) 2  B) 4  C) 6  D) 8

5. An auto repair shop charges $36 per hour, plus the cost of replaced parts. Which of the following expressions can be used to calculate the total cost of repairing a car, where \( h \) represents the number of hours of work and the cost of replaced parts is $85?  (Lesson 1-1)
   
   A) \( 36 + h + 85 \)  B) \( (85 \times h) + 36 \)
   C) \( 36 + 85 \times h \)  D) \( (36 \times h) + 85 \)

6. Which expression is equivalent to \( 3(2x + 3) + 2(x + 1)? \)  (Lessons 1-5 and 1-6)
   
   A) \( 7x + 8 \)  B) \( 8x + 4 \)  C) \( 8x + 9 \)  D) \( 8x + 11 \)

7. Find a counterexample for the following statement. (Lesson 1-7)
   If \( x \) is a positive integer, then \( x^2 \) is divisible by 2.
   
   A) 2  B) 3  C) 4  D) 6

8. The circle graph shows the regions of birth of foreign-born persons in the United States in 2000. According to the graph, which statement is not true? (Lesson 1-9)

   Central America 34.5%  Europe 15.3%  Asia 25.5%  Caribbean 9.9%  South America 6.6%  Other 8.1%

   A) More than \( \frac{1}{3} \) of the foreign-born population is from Central America.
   B) More foreign-born people are from Asia than Central America.
   C) About half of the foreign-born population comes from Central America or Europe.
   D) About half of the foreign-born population comes from Central America, South America, or the Caribbean.

Test-Taking Tip

Questions 1, 3, and 8

Read each question carefully. Be sure you understand what the question asks. Look for words like not, estimate, and approximately.
9. There are 32 students in the class. Five eighths of the students are girls. How many boys are in the class? (Prerequisite Skill)

10. Tonya bought two paperback books. One book cost $8.99 and the other $13.99. Sales tax on her purchase was 6%. How much change should she receive if she gives the clerk $25? (Prerequisite Skill)

11. Refer to the bar graph. In which year was the difference between the number of home runs hit by the two players the least? (Prerequisite Skill)

12. Write a verbal expression for \( \frac{x^2}{y + 5} \). (Lesson 1-1)

13. Write \( 7 \cdot 7 \cdot a \cdot a \cdot a \cdot a \cdot a \) using exponents. (Lesson 1-1)

14. Find the perimeter of the triangle. (Lesson 1-5)

15. The Lee family is going to play miniature golf. The family is composed of two adults and four children. (Lesson 1-3)

<table>
<thead>
<tr>
<th>Greens Fees</th>
<th>before 6 P.M.</th>
<th>after 6 P.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult (a)</td>
<td>$5.00</td>
<td>$6.50</td>
</tr>
<tr>
<td>Children (c)</td>
<td>$3.00</td>
<td>$4.50</td>
</tr>
</tbody>
</table>

a. Write an inequality to show the cost for the family to play miniature golf if they don’t want to spend more than $30.

b. How much will it cost the family to play after 6 P.M.?

c. How much will it cost the family to play before 6 P.M.?

16. Workers are draining water from a pond. They have an old pump and a new pump. The graphs below show how each pump drains water. (Lesson 1-8)

a. Describe how the old and new pumps are different in the amount of water they pump per hour.

b. Draw a graph that shows the gallons pumped per hour by both pumps at the same time.

c. Explain what the graph below tells about how the water is pumped out.