TOMS RIVER MATHEMATICS CURRICULUM

Content Area: Mathematics

Course Title: Mathematics

Grade Level: 6

Unit Plan 1 Number System	**Pacing Guide** 8 Weeks
Unit Plan 2 Geometry	**Pacing Guide** 6 Weeks
Unit Plan 3 Expressions & Equations	**Pacing Guide** 7 Weeks
Unit Plan 4 Ratios & Proportions	**Pacing Guide** 7 Weeks
Unit Plan 5 Statistics & Probability	**Pacing Guide** 6 Weeks

Date Created: **2/06/2014**

Board Approved on:

OCEAN COUNTY MATHEMATICS
Unit Overview

Content Area: Mathematics

Domain: Number System

Cluster:

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Cluster Summary: Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

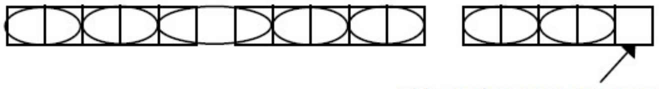
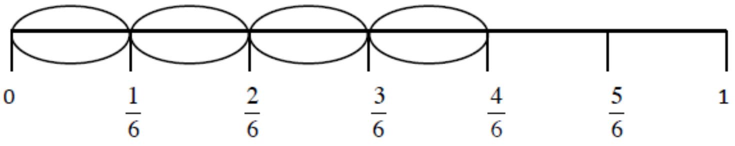
Primary interdisciplinary connections: Science, Social Studies, Language Arts, Technology, and 21st Century Life & Careers (see www.njcccs.org)

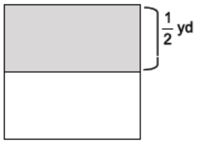
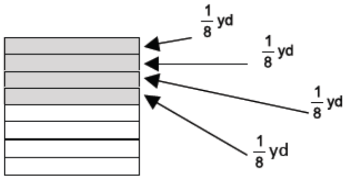
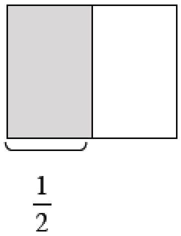
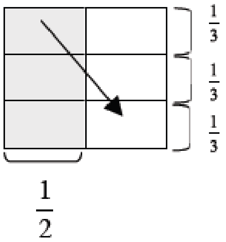
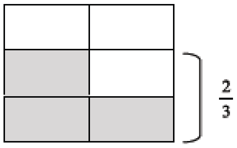
21st century themes: 21st Century Life & Careers; Personal Financial Literacy; Career Awareness, Exploration, & Preparation; Career & Technical Education

Learning Targets

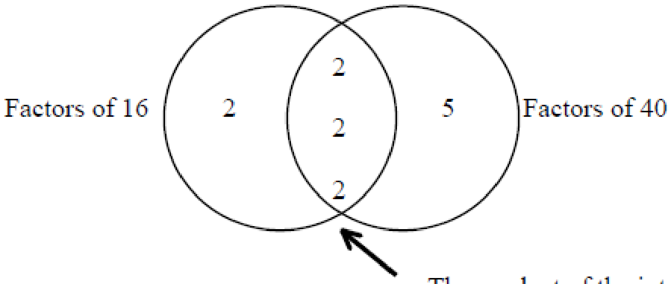
Content Standards

Number	**Common Core Standard for Mastery: Unpacking** **What does this standard mean a student will know and be able to do?** **Source:** North Carolina Department of Public Instruction (2012, August). 6th Grade Mathematics Unpacked Contents. *Public Schools of North Carolina*. Retrieved February 20, 2013, from http://maccss.ncdpi.wikispaces.net/file/view/6th%20grade%20 Mathematics%20unpacked% 20revised.pdf/360876870/6th%20grade%20Mathematics%20u
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3)*	In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division. **Example 1:**

÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	Students understand that a division problem such as 3 ÷ 2/5 is asking, “how many 2/5 are in 3?” One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of ½. Therefore, 3 ÷ 2/5 = 7 ½, meaning there are 7 ½ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.  This section represents one-half of two-fifths Students also write contextual problems for fraction division problems. For example, the problem, 2/3 ÷ 1/6 can be illustrated with the following word problem: **Example 2:** Susan has 2/3 of an hour left to make cards. It takes her about 1/6 of an hour to make each card. About how many can she make? This problem can be modeled using a number line. a. Start with a number line divided into thirds.  b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.  c. Each circled part represents 1/6. There are four sixths in two-thirds; therefore, Susan can make 4 cards. **Example 3:** Michael has of a yard of fabric to make book covers. Each book cover is made from of a yard of fabric. How many book covers can Michael make?

	Solution: Michael can make 4 book covers.   **Example 4:** Represent ½ ÷ $^2/_3$ in a problem context and draw a model to show your solution. **Context:** A recipe requires 2/3 of a cup of yogurt. Rachel has ½ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?    **Explanation of Model:** The first model shows ½ cup. The shaded squares in all three models show the ½ cup. The second model shows ½ cup and also shows 1/3 cups horizontally. The third model shows ½ cup moved to fit in only the area shown by 2/3 of the model. 2/3 is the new referent unit (whole). 3 out of the 4 squares in the 2/3 portion are shaded. A ½ cup is only ¾ of a 2/3 cup portion, so only ¾ of the recipe can be made.
6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.	In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value.

	Example 1: When dividing 32 into 8456, students should say, “there are 200 thirty-twos in 8456” as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64. There are 200 thirty twos in 8456. 200 times 32 is 6400. 8456 minus 6400 is 2056. There are 60 thirty twos in 2056. 60 times 32 is1920. 2056 minus 1920 is 136. There are 4 thirty twos in 136. 4 times 32 is128. The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as 8/32 or ¼. There is ½ of a thirty two in 8. 8456 = 264 * 32 + 8
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	Procedural fluency is defined by the Common Core as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”. In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports student understanding of decimal operations. **Example 1:** First estimate the sum of 12.3 and 9.75. *Solution:* An estimate of the sum would be 12 + 10 or 22. Student could also state if their estimate is high or low. Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole	In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be

numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	found by 1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor. 2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).  The product of the intersecting numbers is the GCF Students also understand that the greatest common factor of two prime numbers is 1. **Example 1:** What is the greatest common factor (GCF) of 18 and 24? *Solution:* $2*3^2$ = 18 and 2^3*3 = 24. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making 2*3 or 6 the GCF. Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions. **Example 2:** Use the greatest common factor and the distributive property

to find the sum of 36 and 8.
36 + 8 = 4 (9) + 4(2)
44 = 4 (9 + 2)
44 = 4 (11)
44 = 44 ✓

Example 3:
Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.
a. What is the greatest number of students that can attend the picnic?
b. How many bags of chips will each student receive?
c. How many hotdogs will each student receive?

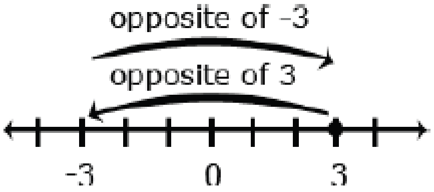
Solution:
a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF).
b. Each student would receive 4 bags of chips.
c. Each student would receive 5 hot dogs.
Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by

1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or
2) using the prime factorization.
    Step 1: find the prime factors of 6 and 8.
    6 = 2 • 3
    8 = 2 • 2 • 2
    Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2
    Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

Example 4:
The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again?

Solution: The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to

	explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20. One way to find the least common multiple is to find the prime factorization of each number: 2^2*5 = 20 and 3*5 = 15. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 (2*2*5). To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5 (2*2*3*5) or 60.
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. **Example 1:** a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids.
6.NS.6.a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.	Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as “the opposite of 4” which would be negative 4. In the example, – (–6.4) would be read as “the opposite of the opposite of 6.4” which would be 6.4. Zero is its own opposite.

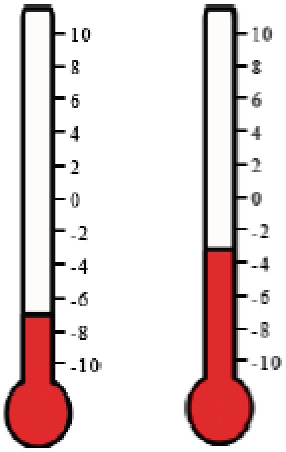
	 **Example 1**: What is the opposite of 2 ½? Explain your answer. *Solution:* - 2 ½ because it is the same distance from 0 on the opposite side.
6.NS.6.b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	Students worked with Quadrant I in elementary school. As the *x*-axis and *y*-axis are extending to include negatives, students begin to work with the Cartesian Coordinate system. Students recognize the point where the *x*-axis and *y*-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +).
6.NS.6.c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the *y* coordinates differ only by signs, which represents a reflection across the *x*-axis. A change is the *x*-coordinates from (-2, 4) to (2, 4), represents a reflection across the *y*-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes. **Example 1:** Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the *x*-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point? $\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, 0.75)$ *Solution:*

	The coordinates of the reflected points would be Note that the *y*-coordinates are opposites. $\left(\frac{1}{2}, 3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, 3\right)$ $(0.25, 0.75)$. **Example 2:** Students place the following numbers on a number line: $-4.5, 2, 3.2, -3\frac{3}{5}, 0.2, -2, \frac{11}{2}$. *Solution:* The numbers in order from least to greatest are: $-4.5, -3\frac{3}{5}, -2, 0.2, 2, 3.2, \frac{11}{2}$ Students place each of these numbers on a number line to justify this order.
6.NS.7 Understand ordering and absolute value of rational numbers.	Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line.
6.NS.7.a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*.	Common models to represent and compare integers include number line models, temperature models and the profit loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. **Operations with integers are not the expectation at this level.** In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

	Case 1: Two positive numbers  5 > 3 5 is greater than 3 3 is less than 5 Case 2: One positive and one negative number  3 > -3 positive 3 is greater than negative 3 negative 3 is less than positive 3 Case 3: Two negative numbers  -3 > -5 negative 3 is greater than negative 5 negative 5 is less than negative 3 **Example 1:** Write a statement to compare – 4 ½ and –2. Explain your answer. *Solution:* – 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.
6.NS.7.b Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C*.	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than". **Example 1:** The balance in Sue's checkbook was –\$12.55. The balance in John's checkbook was –\$10.45. Write an inequality to show the relationship between these amounts. Who owes more? *Solution:* –12.55 < –10.45, Sue owes more than John. The

interpretation could also be “John owes less than Sue”.

Example 2:
One of the thermometers shows -3°C and the other shows -7°C.
Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Solution:
- The thermometer on the left is -7; right is -3
- The left thermometer is colder by 4 degrees
- Either -7 < -3 or -3 > -7

Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3:
A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature:
Albany 5°
Anchorage -6°
Buffalo -7°
Juneau -9°
Reno 12°

Solution:
Juneau -9°
Buffalo -7°

	Anchorage -6° Albany 5° Reno 12°
6.NS.7.c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars*.	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. **Example 1:** Which numbers have an absolute value of 7 *Solution:* 7 and –7 since both numbers have a distance of 7 units from 0 on the number line. **Example 2:** What is the | –3 ½ |? *Solution:* 3 ½ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write | –900| = 900 to describe the distance below sea level.
6.NS.7.d Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars*.	When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases.
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). **Example 1:** What is the distance between (–5, 2) and (–9, 2)? *Solution:* The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both

	coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |). Coordinates could also be in two quadrants and include rational numbers. **Example 2:** What is the distance between (3, –5 ½) and (3, 2 ¼)? *Solution:* The distance between (3, –5 ½) and (3, 2 ¼) would be 7 ¾ units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from –5 ½ to 2 ¼ or by recognizing that the distance (absolute value) from –5 ½ to 0 is 5 ½ units and the distance (absolute value) from 0 to 2 ¼ is 2 ¼ units so the total distance would be 5 ½ + 2 ¼ or 7 ¾ units. Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.
Number	**Common Core Standard for Introduction**
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
Unit Essential Questions • How do operations affect whole numbers? • What makes a computational strategy both effective and efficient? • How is fraction division similar to and different from whole number division? • How are decimal operations similar to and different from	**Unit Enduring Understandings** • The magnitude of numbers affects the outcome of operations on them. • Computational fluency includes understanding the meaning and the appropriate use of numerical operations. • Operations apply to all types of numbers. • Connections exist between pre-fraction skills (GCF, LCM) and fraction operations, enabling fluent & efficient computation. • All numbers have an exact position on the number line.

whole number operations? • What is the difference between factors and multiples? • When do we use least common multiple (LCM) and greatest common factor (GCF)? • How can we apply and extend our understanding of the number line to include negative and opposite numbers?	• All numbers have relationships with other numbers and with zero on the number line.
Unit Objectives *Students will know…* • Procedures for dividing fractions. • Procedures for computing fluently with multi-digit numbers. • Procedures for finding common factors and multiples. • when to use LCM & GCF. • the relationships between numbers on the number line.	**Unit Objectives** *Students will be able to...* • Divide fractions using a standard algorithm and using models. • Divide multi-digit numbers. • Solve real world decimal problems using standard algorithms. • Find the least common multiple (LCM) and greatest common factor (GCF) of a set of numbers. • Use positive and negative numbers and zero to represent real world quantities. • Identify opposite numbers as having opposite signs and being on opposites sides of zero. • Identify, graph, order, and compare integers. • Order rational numbers. • Identify absolute value of numbers as its distance from zero and as a magnitude for a positive or negative number in a real world context. • Compare relative positions of numbers on a number line • Write, interpret, and explain statements of order for rational numbers in real world contexts. • Solve real-world and mathematical problems by graphing points on a coordinate plane.

OCEAN COUNTY MATHEMATICS CURRICULUM
Evidence of Learning

Formative Assessments

- Oral Questioning
- Choral Response
- Partners
- Student Conference
- Self Assessment
- Think-Pair-Share
- Hand Signals
- Peer Reflection
- Communicators
- Graphic Organizers
- Constructive Response
- Teacher Observation
- Exit Card
- Quiz
- Class work
- Math Journals

Summative Assessments

- Quizzes
- Tests
- Projects
- Alternative Assessments
- Benchmark Tests
- Standardized Tests

Modifications (ELLs, Special Education, Gifted and Talented)

- Differentiated Instruction
- Compacting and Extension Activities
- Follow IEP and 504 Plans

Curriculum development Resources/Instructional Materials/Equipment Needed /Teacher Resources:

District Materials

Website Sources:

- Study Island (membership required)
- Skills Tutor (membership required)
- Brain Pop (membership required)
- www.khanacademy.org
- www.aims.edu
- www.aaamath.com
- www.ixl.com
- www.nctm.org
- www.illuminations.nctm.org
- www.nlvm.usu.edu

Teacher Notes:

E10ssential Questions:

- How is division related to realistic situations and to other operations?
- What role does place value play in multi-digit operations?
- How can division be represented and interpreted?
- In what ways can the area of a net be determined?

Unit Enduring Understandings: *Students will understand that …*

- The two types of division – quotative (partitive) and measurement are applied to fractions and decimals as well as to whole numbers.
- Multiplication and division are inverse operations.
- The relationship of the location of the digits and the value of the digits is part of understanding multi-digit operations.
- Division can be represented using multiple formats (manipulatives, diagrams, real-life situations, equations).
- Operations on decimals and whole numbers are based upon place value relationships.
- Problems of area of polygons can be solved by composing and decomposing the polygons.

Unit Objectives: *Students will know…*

- Standard algorithms for addition, subtraction, multiplication and division of multi-digit decimals

OCEAN COUNTY MATHEMATICS
Unit Overview

Content Area: Math

Domain: Geometry

Cluster: Solve real-world and mathematical problems involving area, volume, and surface area.

Cluster Summary: Students build on their work with area in elementary school by reasoning about relationships among shapes to determine are surface area, and volume. Try to find the areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop. And justify formulas for areas of triangle and parallelograms. Students find the areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

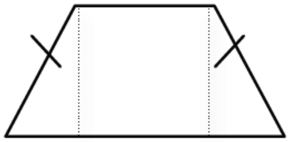
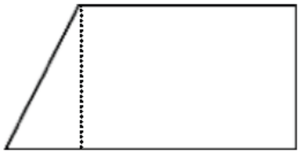
Primary interdisciplinary connections: Science, Social Studies, Language Arts, Technology, and 21st Century Life & Careers (see www.njcccs.org)

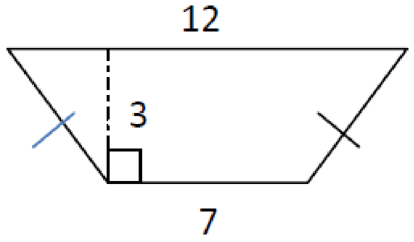
21st century themes: 21st Century Life & Careers; Personal Financial Literacy; Career Awareness, Exploration, & Preparation; Career & Technical Education

Learning Targets

Content Statements

Number	**Common Core Standard for Mastery: Unpacking** **What does this standard mean a student will know and be able to do?** **Source:** North Carolina Department of Public Instruction (2012, August). 6th Grade Mathematics Unpacked Contents. *Public Schools of North Carolina*. Retrieved February 20, 2013, from http://maccss.ncdpi.wikispaces.net/file/view/6th%20grade%20Mathematics%20unpacked%20revised.pdf/360876870/6th%20grade%20Mathematics%20unpacked%20revised.pdf
6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world	Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. “Knowing the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students. Finding the area of triangles is introduced in relationship to the

and mathematical problems.	area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2. The following site helps students to discover the area formula of triangles. http://illuminations.nctm.org/LessonDetail.aspx?ID=L577 Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.  Isosceles trapezoid  Right trapezoid **Note:** Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure. **Example 1:** Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5. *Solution:* Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using: A = ½ bh A = ½ (3 units)(4 units) A = ½ (12 units^2) A = 6 units^2 **Example 2:** Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:
The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2.
The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there are a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units) (3units) or 3.75 units^2.

Using this information, the area of the trapezoid would be:

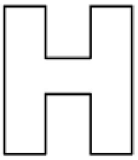
$$\begin{array}{r} 21 \text{ units}^2 \\ 3.75 \text{ units}^2 \\ \underline{+\ 3.75 \text{ units}^2} \\ 28.5 \text{ units}^2 \end{array}$$

Example 3:
A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

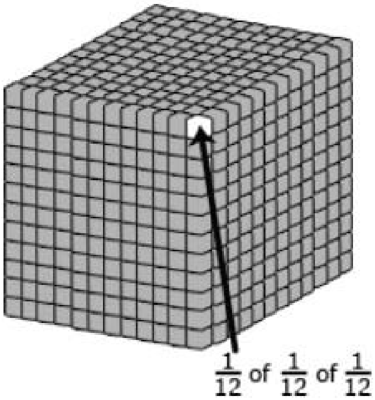
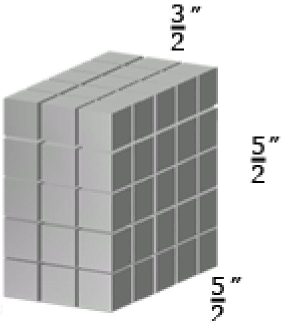
Solution:
The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled). Students may also create a drawing to show this visually.

Example 4:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution:
Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 12 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

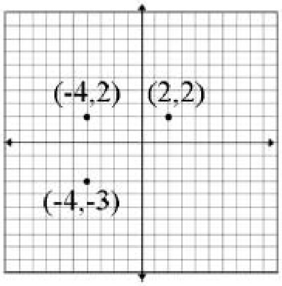
	Example 5: The sixth grade class at Hernandez School is building a giant wooden H for their school. The “H” will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet. 1. How large will the H be if measured in square feet? 2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?  *Solution:* 1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces. The area of the “H” would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2. A second solution would be to decompose the “H” into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the “H” would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2. 2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut **two pieces of wood in half to create four pieces 5 ft. by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5ft. by 2.5 ft.** to create the middle piece. **Example 6:** A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border? *Solution:* Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2 Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2 Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2 The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or **44ft^2**.
6.G.2 Find the volume of a	Previously students calculated the volume of right rectangular

right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (ie. ½ • ½ • ½) Students find the volume of the right rectangular prism with these unit cubes. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (). In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes. **Example 1:** A right rectangular prism has edges of 1 ¼ ", 1", and 1 ½". How many cubes with side lengths of ¼ would be needed to fill the prism? What is the volume of the prism? *Solution:* The number of ¼ " cubes can be found by recognizing the smaller cubes would be ¼" on all edges, changing the dimensions to $^{5}/_{4}$", $^{4}/_{4}$", and $^{6}/_{4}$". The number of one-fourth inch unit cubes making up the prism is 120 (5 x 4 x 6). Each smaller cube has a volume of $^{1}/_{64}$ ($^{1}/_{4}$" x ¼" x ¼"), meaning 64 small cubes would make up the unit cube. Therefore, the volume is $^{5}/_{4}$ x $^{6}/_{4}$ x $^{4}/_{4}$ or $^{120}/_{64}$ (120 smaller cubes with volumes of $^{1}/_{64}$ of 1 $^{56}/_{64}$ -- 1 unit cube with 56 smaller cubes with a volume of $^{1}/_{64}$. **Example 2:** The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $^{1}/_{12}\text{ft}^{3}$

	 **Example 3:** The model shows a rectangular prism with dimensions $^{3}/_{2}$, $^{5}/_{2}$, and $^{5}/_{2}$ inches. Each of the cubic units in the model is $^{1}/_{2}$in. on each side. Students work with the model to illustrate $^{3}/_{2}$ x $^{5}/_{2}$ x $^{5}/_{2}$ = (3 x 5 x 5) x $^{1}/_{8}$. Students reason that a small cube has volume of $^{1}/_{8}$ in^{3} because 8 of them fit in a unit cube. 
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane. This standard can be taught in conjunction with **6.G.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is ½.

Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle.

Example 1:
If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.

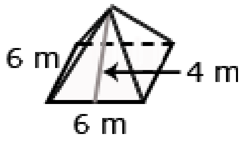
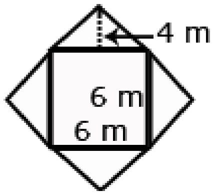


Solution:
To determine the distance along the *x*-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3).
The area would be 5 x 6 or 30 units^2.
The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2:
On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.
1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution:
1. The distance from the library to city hall is 2 miles. The

	coordinates of these buildings have the same *y*-coordinate. The distance between the *x*-coordinates is 2 (from -2 to 0). 2. The three locations form a right triangle. The area is 2 mi^2.
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	A net is a two-dimensional representation of a three-dimensional figure. Students represent three dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area. Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205). Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure. **Example 1:** Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? **Example 2:** Create the net for a given prism or pyramid, and then use the net to calculate the surface area.  

Number	**Common Core Standard for Introduction**
7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
Unit Essential Questions • How can measurements and geometric relationships be used to solve problems? • How does coordinate geometry illustrate a connection between geometry and algebra?	**Unit Enduring Understandings** • Measurements can be used to describe, compare, and make sense of real-world situations, including area, volume, and surface area. • Geometric properties can be used to construct geometric figures. • Coordinate geometry facilitates the visualization of algebraic relationships.
Unit Objectives *Students will know ...* • How to find the area of polygons. • How to find the volume of rectangular prisms. • How to draw polygons on a coordinate plane. • How to use nets to determine surface area.	**Unit Objectives** *Students will be able to…* • Find the area of triangles. • Find the area of quadrilaterals. • Find the area of composite figures. • Solve real-world problems using area. • Find the volume of rectangular prisms. • Show volume is V=Bh and V=lwh. • Solve real world problems using volume. • Draw polygons on a coordinate plane given coordinate vertices. • Solve real world problems using coordinate geometry. • Make a net of a3-D figure. • Identify a 3-D figure from a net. • Use nets to find surface area. • Solve real-world problems using nets.

Unit Enduring Understandings: *Students will understand that …*

- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Properties of 2-dimensional shapes are used in solving problems involving 3-dimensional shapes.
- The value of numbers and application of properties are used to solve problems about our world.

Unit Essential Questions:

- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- What is the relationship between 2-dimensional shapes, 3-dimensional shapes and our world?

Unit Objectives: *Students will know…*

- Formula for volume of a right rectangular prism.
- Procedures for finding surface area of pyramids and prisms

OCEAN COUNTY MATHEMATICS CURRICULUM
Evidence of Learning

Formative Assessments

- Oral Questioning
- Choral Response
- Partners
- Student Conference
- Self Assessment
- Think-Pair-Share
- Hand Signals
- Peer Reflection
- Communicators
- Graphic Organizers
- Constructive Response
- Teacher Observation
- Exit Card
- Quiz
- Class work
- Math Journals

Summative Assessments

- Quizzes
- Tests
- Projects
- Alternative Assessments
- Benchmark Tests
- Standardized Tests

Modifications (ELLs, Special Education, Gifted and Talented)

- Differentiated Instruction
- Compacting and Extension Activities
- Follow IEP and 504 Plans

Curriculum development Resources/Instructional Materials/Equipment Needed /Teacher Resources:

District Materials

Website Sources:

- Study Island (membership required)
- Skills Tutor (membership required)
- Brain Pop (membership required)
- www.khanacademy.org
- www.aims.edu
- www.aaamath.com
- www.ixl.com
- www.nctm.org
- www.illuminations.nctm.org
- www.nlvm.usu.edu

Teacher Notes:

OCEAN COUNTY MATHEMATICS
Unit Overview

Content Area: Mathematics

Domain: Expressions and Equations

Cluster:

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Cluster Summary: Students understand the use of variables in mathematical expressions.

They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

Primary interdisciplinary connections: Science, Social Studies, Language Arts, Technology, and 21st Century Life & Careers (see www.njccs.org)

21st century themes: Century Life & Careers; Personal Financial Literacy; Career Awareness, Exploration, & Preparation; Career & Technical Education

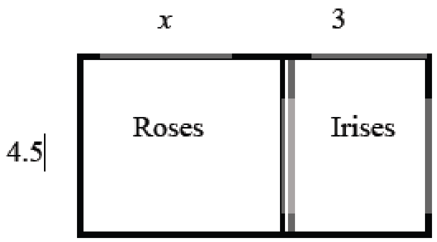
Content Statements

Number	**Common Core Standard for Mastery: Unpacking** **What does this standard mean a student will know and be able to do?** **Source:** North Carolina Department of Public Instruction (2012, August). 6th Grade Mathematics Unpacked Contents. *Public Schools of North Carolina*. Retrieved February 20, 2013, from http://maccss.ncdpi.wikispaces.net/file/view/6th%20grade%20Mathematics%20unpacked%20revised.pdf/360876870/6th%20grade%20Mathematics%20unpacked%20revised.pdf
6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $(1/2)^5$ can be written as 1/2 x 1/2 x 1/2 x 1/2 x 1/2 which has the same value as 1/32). Students recognize that an expression with a variable represents the same mathematics (ie. x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade.

	Order of operations with exponents is the focus in 6th grade. **Example 1:** What is the value of: • 0.2^3 *Solution:* 0.008 • $5 + 2^4 \bullet 6$ *Solution:* 101 • 72 – 24 ÷3 + 26 *Solution:* 67 **Example 2:** What is the area of a square with a side length of $3x$? *Solution:* $3x \bullet 3x = 9x^2$ **Example 3:** $4^x = 64$ *Solution:* $x = 3$ because 4 • 4 • 4 = 64
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers	Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, *n*" could be represented with 5*n* and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is ***important*** for students to read algebraic expressions in a manner that reinforces that the variable represents a number.
6.EE.A.2a Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y.	**Example 1:** Students read algebraic expressions: • *r* + 21 as "some number plus 21" as well as "*r* plus 21" • *n* • 6 as "some number times 6" as well as "*n* times 6" •s/6 and s ÷ 6 as "as some number divided by 6" as well as "*s* divided by 6" **Example 2:** Students write algebraic expressions: • 7 less than 3 times a number *Solution:* $3x – 7$ • 3 times the sum of a number and 5 *Solution:* $3 (x + 5)$ • 7 less than the product of 2 and a number

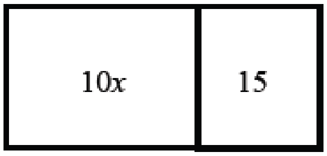
	Solution: $2x – 7$ • Twice the difference between a number and 5 *Solution:* $2(z – 5)$ • The quotient of the sum of x plus 4 and 2 *Solution:* $x + 4/2$
6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms*.	Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent. Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y. There are 4 terms, x^2, $5y$, $3x$, and 6. There are 3 variable terms, x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5y's or $5 \cdot y$. There is one constant term, 6. The expression represents a sum of all four terms.
6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the*	Students evaluate algebraic expressions, using order of operations as needed. Problems such as example 1 below require students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. **Example 1:** Evaluate the expression $3x + 2y$ when x is equal to 4 and y is equal to 2.4. *Solution:* $3 \cdot 4 + 2 \cdot 2.4$

volume and surface area of a cube with sides of length s = 1/2.	12 + 4.8 16.8 **Example 2:** Evaluate 5(*n* + 3) – 7*n*, when *n* = 1/2 *Solution:* 5(1/2 +3) - 7 (1/2) 5(3 1/2) - 3 1/2 Note: 7(1/2) = 7/2 = 3 1/2 17 1/2 - 3 1/2 Students may also reason that 5 groups of 3 1/2 take away 1 group of 3 1/2 would give 4 groups of 3 1/2. Multiply 4 times 3 1/2 to get 14. 14 **Example 3:** Evaluate 7xy when x = 2.5 and y = 9 *Solution:* Students recognize that two or more terms written together indicates multiplication. 7 (2.5) (9) 157.5 In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol. **Example 4:** Evaluate the following expression when x = 4 and y = $2\ \frac{x^2 + y^3}{3}$ *Solution:* $\frac{(4)^2 + (2)^3}{3}$ substitute the values for x and y $\frac{16 + 8}{3}$ raise the numbers to the powers $\frac{24}{3}$ divide 24 by 3 8

	Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. **Example 5:** It costs $100 to rent the skating rink plus $5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25 people? *Solution:* The cost for any number (n) of people could be found by the expression, $100 + 5n$. To find the cost of 25 people substitute 25 in for n and solve to get $100 + 5 * 25 = 225$. **Example 6:** The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25. *Solution*: Substitute 25 in for c and use order of operations to simplify $c + 0.07c$ $25 + 0.07 (25)$ $25 + 1.75$ 26.75
6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.	Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary students illustrate the distributive property with variables. Properties are introduced throughout elementary grades; however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4). **Example 1:** Given that the width is 4.5 units and the length can be represented by x + 2, the area of the flowers below can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$. 

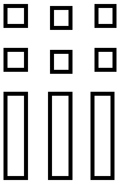
When given an expression representing area, students need to find the factors.

Example 2:
The expression $10x + 15$ can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length $(2x + 3)$. The factors (dimensions) of this figure would be $5(2x + 3)$.



Example 3:
Students use their understanding of multiplication to interpret *$3\,(2 + x)$ as 3 groups of $(2 + x)$.* They use a model to represent x, and make an array to show the meaning of *$3(2 + x)$.* They can explain why it makes sense that *$3(2 + x)$* is equal to *$6 + 3x$.*

An array with 3 columns and $x + 2$ in each column:



Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y ***must be*** *$3y$.* They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$:

Example 4:
Prove that $y + y + y = 3y$
Solution:

$y + y + y$	
$y \cdot 1 + y \cdot 1 + y \cdot 1$	Multiplicative Identity
$y \cdot (1 + 1 + 1)$	Distributive Property
$y \cdot 3$	
$3y$	Commutative Property

Example 5:
Write an equivalent expression for $3(x + 4) + 2(x + 2)$
Solution:

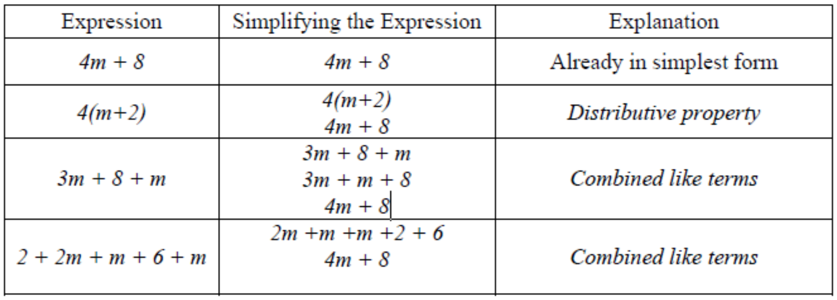
$3(x + 4) + 2(x + 2)$	
$3x + 12 + 2x + 4$	Distributive Property

<table>
<tr><td></td><td>$5x + 16$</td></tr>
<tr><td>6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.</td><td>
Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4x2$ are not like terms since the exponents with the x are not the same. This concept can be illustrated by substituting in a value for x. For example, $9x – 3x = 6x$ not 6. Choosing a value for x, such as 2, can prove non-equivalence.

$9(2) – 3(2) = 6(2)$ however $9(2) – 3(2) = 6$
$18 – 6 = 12$ $18– 6 = 6$
$12 = 12$ $12 \neq 6$

Students can also generate equivalent expressions using the associative, commutative, and distributive properties.
They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1:
Are the expressions equivalent? Explain your answer?
$4m + 8$ $4(m+2)$ $3m + 8 + m$
$2 + 2m + m + 6 + m$

Solution:

<table>
<tr><th>Expression</th><th>Simplifying the Expression</th><th>Explanation</th></tr>
<tr><td>$4m + 8$</td><td>$4m + 8$</td><td>Already in simplest form</td></tr>
<tr><td>$4(m+2)$</td><td>$4(m+2)$
$4m + 8$</td><td>Distributive property</td></tr>
<tr><td>$3m + 8 + m$</td><td>$3m + 8 + m$
$3m + m + 8$
$4m + 8$</td><td>Combined like terms</td></tr>
<tr><td>$2 + 2m + m + 6 + m$</td><td>$2m + m + m + 2 + 6$
$4m + 8$</td><td>Combined like terms</td></tr>
</table>
</td></tr>
<tr><td>6.EE.B.5 Understand solving
equation or inequality as a proc
answering a question: which va
from a specified set, if any, mak
equation or inequality true? Use
substitution to determine wheth
given number in a specified set
makes an equation or inequalit</td><td>
In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.

Example 1:
Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation $26 + n = 100$
</td></tr>
</table>

where *n* is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100." Students ask themselves "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations: $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps to find *n* easily.
- Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of *n*
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

<table>
<tr><td colspan="2">100</td></tr>
<tr><td>26</td><td><i>n</i></td></tr>
</table>

Solution:
Students recognize the value of 74 would make a true statement if substituted for the variable.

$26 + n = 100$
$26 + 74 = 100$
$100 = 100$

Example 2:
The equation $0.44\ s = 11$ where *s* represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

Solution:
There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11.

	By substituting 25 in for *s* and then multiplying, I get 11. 0.44(25) = 11 11 = 11 **Example 3:** Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement? *Solution:* Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, 5 3/4, and 200. Given a set of values, students identify the values that make the inequality true.
6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	Students write expressions to represent various real-world situations. **Example 1:** • Write an expression to represent Susan's age in three years, when *a* represents her present age. • Write an expression to represent the number of wheels, *w*, on any number of bicycles. • Write an expression to represent the value of any number of quarters, *q*. *Solutions:* • $a + 3$ • $2n$ • $0.25q$ Given a contextual situation, students define variables and write an expression to represent the situation. **Example 2:** The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people. n = the number of people $100 + 5n$ No solving is expected with this standard; however, 6.EE.2c does address the evaluating of the expressions. Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many

	bracelets as Jane, then Jane has 1/3 the amount of Sally. If *S* represents the number of bracelets Sally has, the 1/3s or s/3 represents the amount Jane has. Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. **Example 3:** • Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. *Solution:* $2c + 3$ where c represents the number of crayons that Elizabeth has • An amusement park charges \$28 to enter and \$0.35 per ticket. Write an algebraic expression to represent the total amount spent. *Solution:* $28 + 0.35t$ where t represents the number of tickets purchased • Andrew has a summer job doing yard work. He is paid \$15 per hour and a \$20 bonus when he completes the yard. He was paid \$85 for completing one yard. Write an equation to represent the amount of money he earned. *Solution:* $15h + 20 = 85$ where h is the number of hours worked • Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item *Possible solution:* Sarah spent \$15 at a craft store. o She bought one notebook for \$3. o She bought 2 paintbrushes for x dollars. If each paintbrush cost the same amount, what was the cost of one brush? • Bill earned \$5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. *Solution:* \$5.00 + n
6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in	Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer;

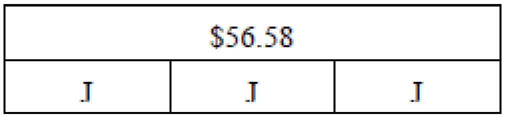
<table>
<tr>
<td>which *p*, *q* and *x* are all nonnegative rational numbers.</td>
<td>however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions.

Students recognize that dividing by 6 and multiplying by 1/6 produces the same result. For example, $x/6 = 9$ and $1/6x = 9$ will produce the same result.

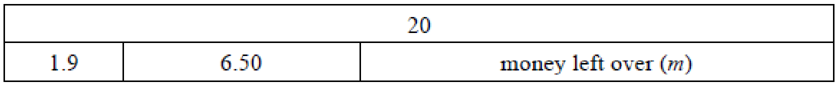
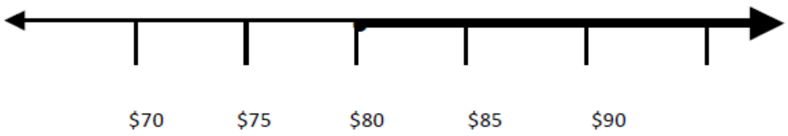
Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem.

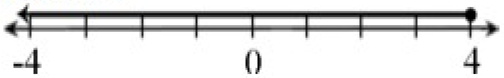
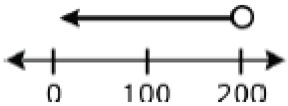
Example 1:
Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.


<table>
<tr><td colspan="3">$56.58</td></tr>
<tr><td>J</td><td>J</td><td>J</td></tr>
</table>

Sample Solution:
Students might say: “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled *J* is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That’s $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58.”

Example 2:
Julie gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.</td>
</tr>
</table>

	 *<u>Solution:</u>* 20 = 1.99 + 6.50 + x, x = $11.51
6.EE.B.8 Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations. **Example 1:** The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, *m*, the class still needs to raise. Represent this inequality on a number line. *Solution:* The inequality *m* ≥ $80 represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.  A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions. **Example 2:** Graph *x* ≤ 4. *Solution:*

	Solution:  **Example 3:** The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line. *Solution:* 200 > *x*, where *x* is the amount spent on groceries. 
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the *x*-axis; the dependent variable is graphed on the *y*-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the *x* variable increases, how does the *y* variable change?) *Relationships should be proportional with the line passing through the origin*. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values. Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students

understand that each form represents the same relationship and provides a different perspective.

Example 1:
What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

Solution:
$y = 2.5x$

Unit Essential Questions

- How can mathematical situations be translated and represented abstractly using variables, expressions, and equations?
- How can patterns be used to identify a relationship between two quantities?
- What are algebraic expressions and how can they be written and evaluated?
- How can equations be graphed?

How can we use rules of arithmetic, algebra, and equivalence to transform equations into equations that can be solved?

Unit Enduring Understandings

- Some mathematical situations can be translated and represented using a variable in an algebraic expression.
- The value of an algebraic expression can be found by replacing the variable(s) with given number(s) and doing the calculation that results.
- There is an agreed upon order in which operations are carried out in a numerical expressions.
- The Distributive Property of Multiplication over Addition lets you multiply a sum by multiplying each addend separately and then finding the sum of the products.
- Some quantities have a mathematical relationship; the value of one quantity can be found if you know the value of the other quantity. Patterns can sometimes be used to identify a relationship between two quantities.
- Some problems can be solved by recording and organizing data in a table and by finding and using numerical patterns in the table.
- Equations can be transformed into equivalent equations and solved using properties of equality and inverse operations.

A solution to an inequality is a value that makes the inequality true.

Unit Objectives
Students will know...

- Write and evaluate algebraic expressions
- Write and evaluate one-variable equations and inequalities

Represent and analyze quantitative relationships

Unit Objectives
Students will be able to…

- Write and evaluate numerical expressions involving whole number exponents
- Write expressions using numbers and variables
- Identify parts of an expression
- Evaluate expressions given specific values for variables
- Solve simple equations using order of operations

between dependent and independent	• Apply the distributive property to generate equivalent expressions • Identify when two expressions are equivalent • Solve an equation or inequality by finding all the values that make it true • Use variables to represent unknown numbers when solving real-world mathematical problems • Solve real-world problems by writing and solving equations • Write, solve, and graph inequalities in real world and mathematical problems • Use graphs, tables, and equations to identify the relationships between dependent and independent variables

OCEAN COUNTY MATHEMATICS CURRICULUM

Evidence of Learning

Formative Assessments

- Oral Questioning
- Choral Response
- Partners
- Student Conference
- Self Assessment
- Think-Pair-Share
- Hand Signals
- Peer Reflection
- Communicators
- Graphic Organizers
- Constructive Response
- Teacher Observation
- Exit Card
- Quiz
- Class work
- Math Journals

Summative Assessments

- Quizzes
- Tests
- Projects
- Alternative Assessments
- Benchmark Tests
- Standardized Tests

Modifications (ELLs, Special Education, Gifted and Talented)

- Differentiated Instruction
- Compacting and Extension Activities
- Follow IEP and 504 Plans

Curriculum development Resources/Instructional Materials/Equipment Needed /Teacher Resources:

District Materials

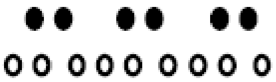
Website Sources:

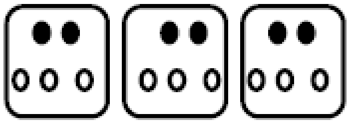
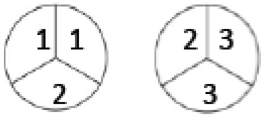
- Study Island (membership required)
- Skills Tutor (membership required)
- Brain Pop (membership required)
- www.khanacademy.org
- www.aims.edu
- www.aaamath.com
- www.ixl.com
- www.nctm.org
- www.illuminations.nctm.org
- www.nlvm.usu.edu

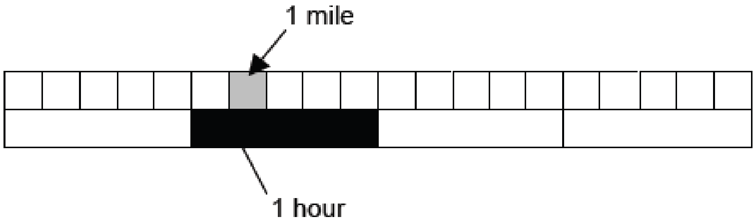
Teacher Notes:

OCEAN COUNTY MATHEMATICS
Unit Overview MATHEMATICS
Unit Overview

<table>
<tr><td colspan="2">Content Area: Mathematics</td></tr>
<tr><td colspan="2">Domain: Ratios and Proportions</td></tr>
<tr><td colspan="2">Cluster: Understand ratio concepts and use ratio reasoning to solve problems</td></tr>
<tr><td colspan="2">Cluster Summary: Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus, students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
Primary interdisciplinary connections: Science, Social Studies, Language Arts, Technology, and 21st Century Life & Careers (see www.njccs.org)
21st century themes: 21st Century Life & Careers; Personal Financial Literacy; Career Awareness, Exploration, & Preparation; Career & Technical Education</td></tr>
<tr><td colspan="2" align="center">Learning Targets</td></tr>
<tr><td colspan="2">Content Statements</td></tr>
<tr><td>Number</td><td>Common Core Standard for Mastery: Unpacking
What does this standard mean a student will know and be able to do?
Source: North Carolina Department of Public Instruction (2012, August). 6th Grade Mathematics Unpacked Contents. <i>Public Schools of North Carolina</i>. Retrieved February 20, 2013, from http://maccss.ncdpi.wikispaces.net/file/view/6th%20grade%20Mathematics%20unpacked%20revised.pdf/360876870/6th%20grade%20Mathematics%20unpacked%20revised.pdf</td></tr>
<tr><td>6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</td><td>A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish).

Example 1:
A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: 6/9, 6 to 9 or 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as

These values can be regrouped into 2 black circles (guppies) to 3 white circles (goldfish), which would reduce the ratio to, 2/3, 2 to 3 or 2:3.</td></tr>
</table>

	 Students should be able to identify and describe any ratio using "For every _____, there are _____" In the example above, the ratio could be expressed saying, "For every 2 guppies, there are 3 goldfish". **NOTE**: Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. For example, ratios are often used to make "part-part" comparisons but fractions are not.
6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."	A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time. Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers. **Example 1:** There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate) *Solution:* This can be modeled as shown below to show that there is 2/3 of a cookie for 1 student, so the unit rate is 2/3: 1.  **Example 2:** On a bicycle Jack can travel 20 miles in 4 hours. What are the

	unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)? *Solution:* Jack can travel 5 miles in 1 hour written as 5 mi/ 1 hr and it takes 1/5 of an hour to travel each mile written as 1/5 hr / 1 mi. Students can represent the relationship between 20 miles and 4 hours. 
6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. Scaling up or down with multiplication maintains the equivalence. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, ***whole number*** measurements are the expectation for this standard.
6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	**Example 1:** At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54. *Solution:* To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. 1 • 7 = 7; 6 • 7 = 42). Red numbers indicate solutions.

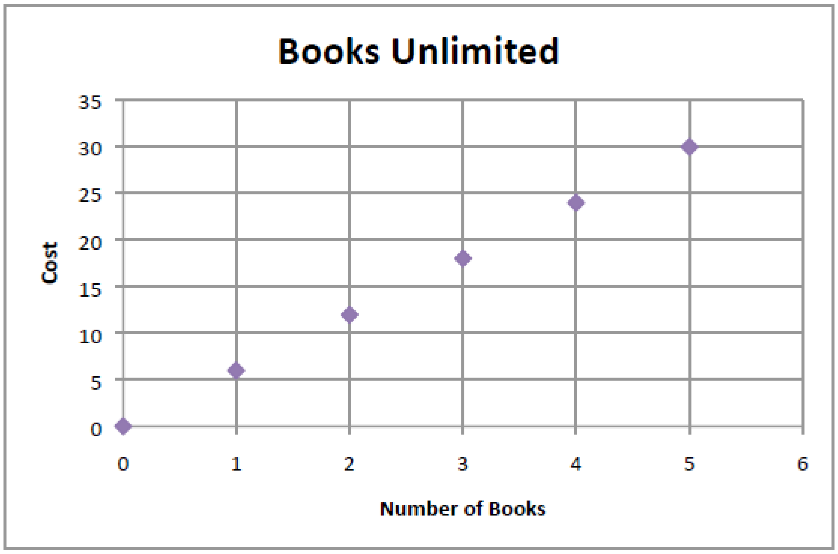
Number of Books (n)	**Cost (C)**
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	**Cost (C)**
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be
$C = 6n$, while the equation for the second bookstore is $C = 5n$. The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2:
Ratios can also be used in problem solving by thinking about the total amount for each ratio unit. The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:
There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet;

therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example 4:
Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:
There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

Example 1:
In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

Peanuts	Chocolate
3	2

Solution:
One possible solution is for students to find the number of cups

<table>
<tr>
<td></td>
<td>of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving 2/3 cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine (9 •2/3), giving 6 cups of chocolate.

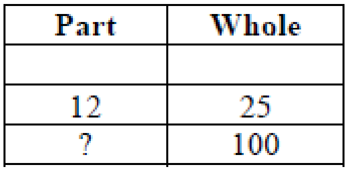
Example 2:
If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

<i>Solution:</i>
The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.</td>
</tr>
<tr>
<td>6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</td>
<td>This is the students’ first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.

Students use ratios to identify percents.

Example 1:
What percent is 12 out of 25?

<i>Solution:</i> One possible solution method is to set up a ratio table:
Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.


<table>
<tr><th>Part</th><th>Whole</th></tr>
<tr><td></td><td></td></tr>
<tr><td>12</td><td>25</td></tr>
<tr><td>?</td><td>100</td></tr>
</table>

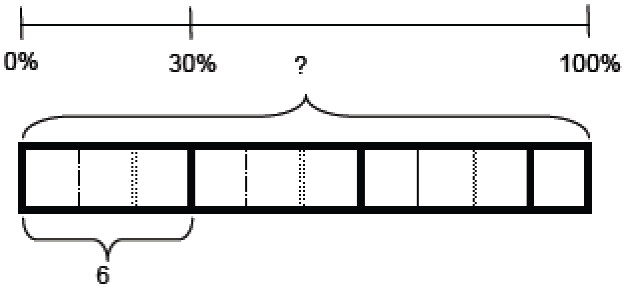
Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Example 2:
What is 40% of 30?
<i>Solution:</i> There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.</td>
</tr>
</table>

See the weblink below for more information. http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Students also determine the whole amount, given a part and the percent.

Example 3:
If 30% of the students in Mrs. Rutherford’s class like chocolate ice cream, then how many students are in Mrs. Rutherford’s class if 6 like chocolate ice cream?



Solution: 20

Example 4:
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?

Solution:

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

6.RP.A.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, 12 inches/ 1 foot is a conversion factor since the numerator and denominator equals the same amount. Since the ratio is

	equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as 1 foot/ 12 inches allowing for the conversion ratios to be expressed in a format so that units will “cancel”. Students use ratios as conversion factors and the identity property for multiplication to convert ratio units. **Example 1:** How many centimeters are in 7 feet, given that 1inch ≈ 2.54 cm. *Solution:* $7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \cancel{\text{feet}} \times \frac{12 \cancel{\text{inches}}}{1 \cancel{\text{foot}}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{inch}}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$ **Note:** Conversion factors will be given. Conversions can occur both between and across the metric and English systems. Estimates are not expected.
Unit Essential Questions • What are ratios and rates and how are they used in solving problems? • What is a proportion and what role does a ratio play in a proportion? • When are situations proportional? • How can numbers, expressions, measures, and objects be compared to other numbers, expressions, measures, and objects? • What are the different ways mathematics content and practices can be applied to solve problems?	**Unit Enduring Understandings** • A ratio is a special relationship between two quantities where for every x units of one quantity there are y units of another quantity. The quantities being compared in a ratio are called terms. • In a proportional relationship there are an infinite number of ratios equal to the lowest terms or constant ratio. Equal ratios can be found by multiplying both terms by the same non-zero number. • A unit rate is a rate that compares a quantity to one unit of another quantity. • A formula is a common relationship between quantities expressed as an equation. • A special proportional relationship involves distance (d), rate (r), and time (t). The formula showing this relationship is d= r x t. • Rates are easily compared when each is expressed as a unit rate.
Unit Objectives *Students will know...* • Understand the concept of a ratio • Understand the concept of a	**Unit Objectives** *Students will be able to…* • Use ratio language to describe a relationship between two quantities • Express ratios in three ways (a/b a to b and a:b) when

unit rate • Use ratios and rates to solve real world problems	b≠ 0 • Find the unit rate • Make tables and graphs to represent equivalent ratios • Solve unit rate problems including pricing and constant speed • Find the percent of a quantity as a rate per 100 • Determine what percent one number is to another • Use ratio reasoning to convert measurement units

OCEAN COUNTY MATHEMATICS CURRICULUM
Evidence of Learning

Formative Assessments

- Oral Questioning
- Choral Response
- Partners
- Student Conference
- Self Assessment
- Think-Pair-Share
- Hand Signals
- Peer Reflection
- Communicators
- Graphic Organizers
- Constructive Response
- Teacher Observation
- Exit Card
- Quiz
- Class work
- Math Journals

Summative Assessments

- Quizzes
- Tests
- Projects
- Alternative Assessments
- Benchmark Tests
- Standardized Tests

Modifications (ELLs, Special Education, Gifted and Talented)

- Differentiated Instruction
- Compacting and Extension Activities
- Follow IEP and 504 Plans

Curriculum development Resources/Instructional Materials/Equipment Needed /Teacher Resources:

District Materials

Website Sources:

- Study Island (membership required)
- Skills Tutor (membership required)
- Brain Pop (membership required)
- www.khanacademy.org
- www.aims.edu
- www.aaamath.com
- www.ixl.com
- www.nctm.org
- www.illuminations.nctm.org
- www.nlvm.usu.edu

Teacher Notes:

Unit Enduring Understandings: *Students will understand that …*

- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percents.
- A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates.)
- A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.
- Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.).

Unit Essential Questions:

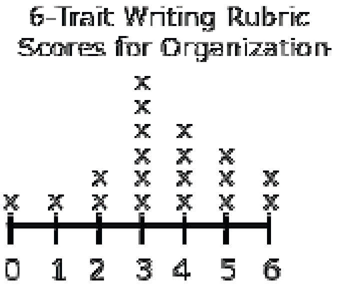
- When is it useful to be able to relate one quantity to another?
- How are ratios and rates similar and different?
- What is the connection between a ratio and a fraction?

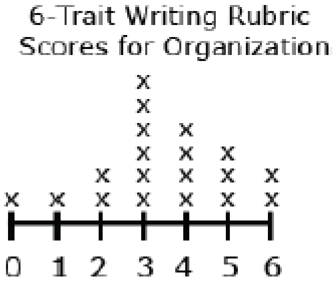
Unit Objectives: Students will know…

- A ratio compares two related quantities.
- Ratios can be represented in a variety of formats including each, to, per, for each, %, 1/5, etc.
- A percent is a type of ratio that compares a quantity to 100.
- A unit rate is the ratio of two measurements in which the second term is 1.
- When it is appropriate to use ratios/rates to solve mathematical or real life problems.
- Mathematical strategies for solving problems involving ratios and rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

Content Area: Mathematics
Domain: Statistics & Probability
Cluster: • Develop understanding of statistical variability • Summarize and describe distributions
Cluster Summary: Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. **Primary interdisciplinary connections:** Science, Social Studies, Language Arts, Technology, and 21st Century Life & Careers (see www.njcccs.org) **21st century themes:** 21st Century Life & Careers; Personal Financial Literacy; Career Awareness, Exploration, & Preparation; Career & Technical Education
Learning Targets
Content Statements

Number	**Common Core Standard for Mastery: Unpacking** **What does this standard mean a student will know and be able to do?** **Source:** North Carolina Department of Public Instruction (2012, August). 6th Grade Mathematics Unpacked Contents. *Public Schools of North Carolina*. Retrieved February 20, 2013, from http://maccss.ncdpi.wikispaces.net/file/view/6th%20grade%20M athematics%20unpacked%20revised.pdf/360876870/6th%20grade%20Mathematics%20unpacked%20revised.pdf
6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages*.	Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates

	variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?”
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots. **Example 1:** The dot plot shows the writing scores for a group of students on organization. Describe the data.  *Solution:* The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68. NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (i.e. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic. **Example 1:** Consider the data shown in the dot plot of the six trait scores for organization for a group of students. • How many students are represented in the data set? • What are the mean and median of the data set? What do these values mean? How do they compare? • What is the range of the data? What does this value mean?  6-Trait Writing Rubric Scores for Organization *Solution:* • 19 students are represented in the data set. • The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower. • The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.
6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Students display data graphically using number lines. Dot plots, histograms and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others. Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

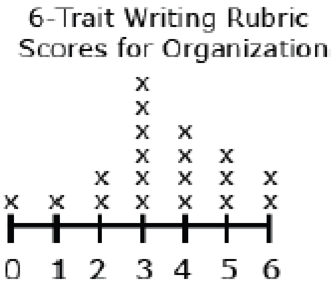
A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.

Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.
Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Example 1:
Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0,1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

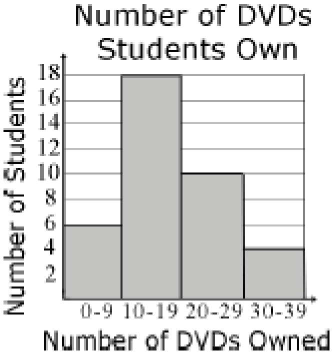
Solution:



Example 2:
Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

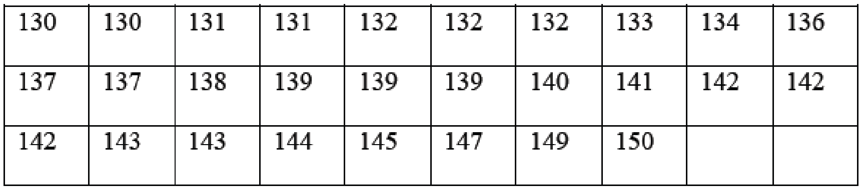
11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:
A histogram using 5 intervals (bins) 0-9, 10-19, …30-39) to organize the data is displayed below.

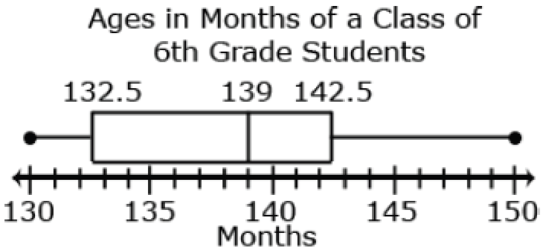


Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:
Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

<table>
<tr>
<td></td>
<td>

<table>
<tr><td>130</td><td>130</td><td>131</td><td>131</td><td>132</td><td>132</td><td>132</td><td>133</td><td>134</td><td>136</td></tr>
<tr><td>137</td><td>137</td><td>138</td><td>139</td><td>139</td><td>139</td><td>140</td><td>141</td><td>142</td><td>142</td></tr>
<tr><td>142</td><td>143</td><td>143</td><td>144</td><td>145</td><td>147</td><td>149</td><td>150</td><td></td><td></td></tr>
</table>

<i>Solution:</i>
Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



This box plot shows that
• ¼ of the students in the class are from 130 to 132.5 months old
• ¼ of the students in the class are from 142.5 months to 150 months old
• ½ of the class are from 132.5 to 142.5 months old
• The median class age is 139 months.
</td>
</tr>
<tr>
<td>6.SP.5 Summarize numerical data sets in relation to their context, such as by:</td>
<td>Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles.</td>
</tr>
<tr>
<td>6.SP.5a Reporting the number of observations.</td>
<td>Students record the number of observations.</td>
</tr>
</table>

6.SP.5b Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable).
6.SP.5c Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	Measures of Center Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point). Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average. **Example 1:** Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 Project 4: ?? What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning. *Solution:* One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

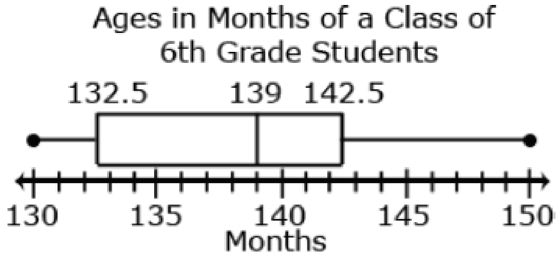
Measures of Variability

Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:

What is the IQR of the data below:



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2:

The following data set represents the size of 9 families:

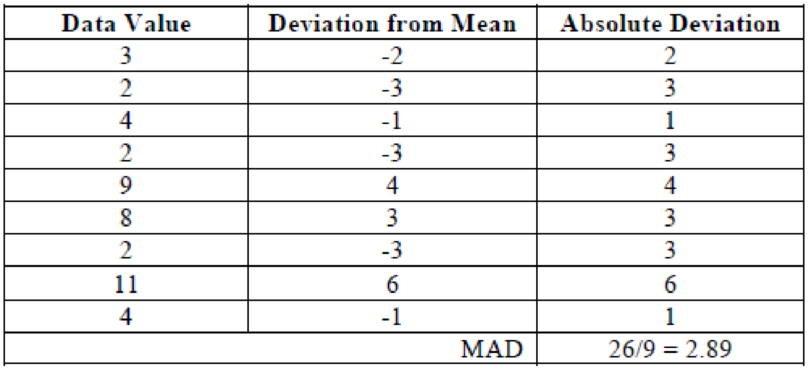
3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1

<table>
<tr>
<td></td>
<td>3. Find the average of these absolute deviations.
The table below shows these calculations:
<table>
<tr><th>Data Value</th><th>Deviation from Mean</th><th>Absolute Deviation</th></tr>
<tr><td>3</td><td>-2</td><td>2</td></tr>
<tr><td>2</td><td>-3</td><td>3</td></tr>
<tr><td>4</td><td>-1</td><td>1</td></tr>
<tr><td>2</td><td>-3</td><td>3</td></tr>
<tr><td>9</td><td>4</td><td>4</td></tr>
<tr><td>8</td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>-3</td><td>3</td></tr>
<tr><td>11</td><td>6</td><td>6</td></tr>
<tr><td>4</td><td>-1</td><td>1</td></tr>
<tr><td colspan="2">MAD</td><td>26/9 = 2.89</td></tr>
</table>This value indicates that on average family size varies 2.89 from the mean of 5.

Students understand how the measures of center and measures of variability are represented by graphical displays.</td>
</tr>
<tr>
<td>6.SP.5d Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</td>
<td>Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.</td>
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Number	**Common Core Standard for Introduction**
7.SP.C	Investigate chance processes and develop, use, and evaluate probability models.
Unit Essential Questions • What are the different ways that data can be represented? • What are the different numerical measures that describe data sets? • How do you determine which numerical measure is the most appropriate to use to analyze a given data set?	**Unit Enduring Understandings** • Statistical questions anticipate variability in the data. These questions can be answered by collecting and analyzing data. The question to be answered determines the data that needs to be collected. • Each type of graph is most appropriate for certain kinds of data. A histogram uses bars to compare continuous numerical data grouped into intervals. • Box plots are useful for plotting data above a number line. Box plots show the spread for each quarter of the data. • A set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.
Unit Objectives *Students will know ...* • The concept of statistical variability • How to find measures of central tendencies • How to summarize and describe data distributions through graphing	**Unit Objectives** *Students will be able to…* • Determine whether a question is a statistical question or not. • Describe data sets by looking at their center, spread, and overall shape. • Find the mean of a data set. • Find the median, mode, and range of data sets. • Make and use histograms, dot plots, and box plots. • Summarize numerical data sets by identifying sample size, possible bias, and units of measurement. • Use mean absolute deviation and interquartile range to measure variability in a set of data. • Decide which measure of central tendency most accurately describes a given set of data • Recognize an appropriate statistical measures.

OCEAN COUNTY MATHEMATICS CURRICULUM

Evidence of Learning

Formative Assessments

- Oral Questioning
- Choral Response
- Partners
- Student Conference
- Self Assessment
- Think-Pair-Share
- Hand Signals
- Peer Reflection
- Communicators
- Graphic Organizers
- Constructive Response
- Teacher Observation
- Exit Card
- Quiz
- Class work
- Math Journals

Summative Assessments

- Quizzes
- Tests
- Projects
- Alternative Assessments
- Benchmark Tests
- Standardized Tests

Modifications (ELLs, Special Education, Gifted and Talented)

- Differentiated Instruction
- Compacting and Extension Activities
- Follow IEP and 504 Plans

Curriculum development Resources/Instructional Materials/Equipment Needed /Teacher Resources:

District Materials

Website Sources:

- Study Island (membership required)
- Skills Tutor (membership required)
- Brain Pop (membership required)
- www.khanacademy.org
- www.aims.edu
- www.aaamath.com
- www.ixl.com
- www.nctm.org
- www.illuminations.nctm.org
- www.nlvm.usu.edu

Teacher Notes: