

<b>Original Adoption:</b>	3/8/2016
<b>Revised:</b>	July 2019
<b>Board Approved:</b>	8/21/2019

<b>Toms River Regional Schools Calculus Curriculum</b>	
<b>Content Area: Mathematics</b>	
<b>Course Title: Calculus</b>	<b>Grade Level: High School</b>
Prerequisite Skills/Overview of Calculus	25 Days
Limits and Continuity	20 Days
Derivatives and Their Applications	60 Days
Integrals and Their Applications	65 Days

**Introduction**

**Effective mathematics education provides students with a balanced instructional program. In such a program, students become proficient in basic computational skills and procedures, develop conceptual understandings, and become skilled at problem solving. Standards-based mathematics instruction starts with basic material and increases in scope and content as the years progress.**

**The curriculum is aligned to the NJSL for Mathematics. Activities outlined in this curriculum infuse the Standards for Mathematical Practice. In alignment to the content and practice standards, Calculus students will extend their knowledge of mathematics as they learn to derive and integrate functions and apply these concepts to real world applications. The ideas of limits and continuity will be introduced to further students understanding of the world around them.**

**Students use functions, with their derivatives and antiderivatives to model real world applications and their knowledge of their properties to explain the world around them. Students will use numerical, graphical, and algebraic models to solve problems and present their findings.**

<b>Unit 1: Prerequisite Skills/Overview of Calculus</b>	<b>Duration: 20 Days</b>
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## Standards/Learning Targets

### Focus Standards (Major Standards)

A-APR.6 Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.4 Solve quadratic equations in one variable.

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in

terms of a context.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF.1 Write a function that describes a relationship between two quantities.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive

and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.4 Find inverse functions.

F-BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

G-MG.3 Apply geometric methods to solve design problems.

### Supporting and Additional Standards

The following Standards for Mathematical Practice and select New Jersey Student Learning Standards should be covered throughout the various units of the curriculum.

#### Standards for Mathematical Practices

- |      |  |   |
|------|--|---|
| MP.1 | Make sense of problems and persevere in solving them | <ul style="list-style-type: none"><li>● Find meaning in problems</li><li>● Look for entry points</li><li>● Analyze, conjecture and plan solution pathways</li></ul> |
|------|--|---|

MP.2	Reason abstractly and quantitatively.	<ul style="list-style-type: none"> <li>● Monitor and adjust</li> <li>● Verify answers</li> <li>● Ask themselves the question: “Does this make sense?”</li> <li>● Make sense of quantities and their relationships in problems</li> <li>● Learn to contextualize and de-contextualize</li> <li>● Create coherent representations of problems</li> </ul>
MP.3	Construct viable arguments and critique the reasoning of others.	<ul style="list-style-type: none"> <li>● Understand and use information to construct arguments</li> <li>● Make and explore the truth of conjectures</li> <li>● Recognize and use counterexamples</li> <li>● Justify conclusions and respond to arguments of others</li> </ul>
MP.4	Model with Mathematics.	<ul style="list-style-type: none"> <li>● Apply mathematics to problems in everyday life</li> <li>● Make assumptions and approximations</li> <li>● Identify quantities in a practical situation</li> <li>● Interpret results in the context of the situation and reflect on whether the results make sense</li> </ul>
MP.5	Use appropriate tools strategically.	<ul style="list-style-type: none"> <li>● Consider the available tools when solving problems</li> <li>● Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)</li> <li>● Make sound decisions of which of these tools might be helpful</li> </ul>
MP.6	Attend to precision.	<ul style="list-style-type: none"> <li>● Communicate precisely to others</li> <li>● Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes</li> <li>● Calculate accurately and efficiently</li> </ul>

MP.7 Look for and make use of structure.

- Discern patterns and structures
- Can step back for an overview and shift perspective
- See complicated things as single objects or as being composed of several objects

**Primary Interdisciplinary Connections:** Infused within the unit are connections to the NJSLs for Mathematics, Language Arts Literacy

WHST.11-12.10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

● **TECHNOLOGY STANDARDS and APPLY explicit standards as appropriate.**

- 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
- A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations
- C. Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
- E: Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
- F: Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

**21st Century Themes/Careers: Through instruction in life and career skills, all students acquire the knowledge and skills needed to prepare for life as citizens and workers in the 21st century. For further clarification see NJ World Class Standards at [www.NJ.gov/education/aps/cccs/career/](http://www.NJ.gov/education/aps/cccs/career/)**

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

9.2.8.B.1 Research careers within the 16 Career Clusters and determine attributes of career success.

9.2.12.C.3 Identify transferable career skills and design alternate career plans

### Evidence of Student Learning

#### Performance Tasks/Use of Technology:

- [www.mathxforschool.com](http://www.mathxforschool.com)
- [www.khanacademy.com](http://www.khanacademy.com)
- [www.desmos.com](http://www.desmos.com)
- [www.kahoot.com](http://www.kahoot.com)
- [www.quizizz.com](http://www.quizizz.com)

#### Other Assessments

##### Formative

- Observation
- Homework
- Class Participation
- Whiteboards/communicators
- Think-Pair-Share
- Do-Now
- Notebook Checks
- Writing Prompts
- Exit Tickets
- Classroom Games
- Self-assessment

##### Summative

- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects

	<p><b>Benchmark</b></p> <ul style="list-style-type: none"> <li>• Quarterly Benchmark Assessment</li> <li>• Midterm Assessment</li> </ul> <p><b>Alternative</b></p> <ul style="list-style-type: none"> <li>• Portfolio Project</li> <li>• Modified assignments</li> </ul>
<b>Knowledge and Skills</b>	
<b>Content</b>	<b>Skills</b>
<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>• How to solve equations and inequalities.</li> <li>• How to graph relations and functions.</li> <li>• How to simplify, graph, add, subtract, multiply, and divide rational expressions.</li> <li>• How to understand and use function notation.</li> <li>• How to simplify and evaluate trigonometric expressions.</li> <li>• How to solve and graph trigonometric equations</li> </ul>	<p><i>Students will be able to..</i></p> <ul style="list-style-type: none"> <li>• Solve equations and inequalities</li> <li>• Graph relations and functions</li> <li>• Simplify, graph, add, subtract, multiply, and divide rational expressions.</li> <li>• Understand and use function notation.</li> <li>• Simplify and evaluate trigonometric expressions.</li> <li>• Solve and graph trigonometric equations.</li> </ul>
<b>Instructional Plan</b>	
<b>Suggested Activities</b>	<b>Resources</b>

- Graphing Calculator
- Microsoft Excel/PowerPoint
- Teacher-made tests, worksheets, warm-ups, and quizzes
- Computer software to support unit
- Smart board
- Document camera
- [www.ixl.com](http://www.ixl.com)
- [www.purplemath.com](http://www.purplemath.com)
- [www.brightstorm.com](http://www.brightstorm.com)
- [www.coolmath.com](http://www.coolmath.com)

## MODIFICATIONS

### *English Language Learners*

- Provide clear and specific directions
- Allow for alternate forms of responses- drawing or speaking instead of writing to demonstrate knowledge when you are not specifically assessing writing
- Provide class notes ahead of time to allow students to preview material and increase comprehension
- Provide extended time
- Model directions and provide gestures to increase understanding
- Simplify written and verbal instructions
- Allow the use of an online dictionary to look up the definition and hear the pronunciation of unknown words
- Create a nurturing environment with structured routines
- Teach study skills
- Gather materials such as visuals, models, manipulatives, videos and other tangible referents to contextualize the lesson.

### *Special Education*

- Provide clear and specific directions
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- Provide frequent breaks
- Provide written directions with models and diagrams when possible
- Utilize graphic organizers
- Assign peer tutor
- Provide manipulatives
- Frequently check for understanding
- Provide immediate praise and feedback
- Have student repeat directions to check for understanding
- Create a nurturing environment with structured routines

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*Gifted and Talented*

- Extension activities
- Opportunities for Critical Thinking
- Problem Solving/Design Challenges
- Technology Integration
- Student Choice Activities
- Student Driven Activities
- Group Projects
- Tiered Activities

*Students at Risk of School Failure*

- Extended Time
- Flexible Grouping
- Small Group Instruction
- Peer Buddies
- Graphic Organizers
- Chunking Information
- Scaffolded Questioning
- Tiered Activities
- Manipulatives
- Build Background/Vocabulary
- Math Word Wall/Word Bank
- Modified Assignments
- Gradual Release Model
- Preferential Seating
- Visual Cues/Models
- Technology Integration

- Assistive Technology

**Core Instructional and Supplemental Materials**

- Calculus Text
- [www.kutasoftware.com](http://www.kutasoftware.com)
- Text Support Materials

**Teacher Notes:**

<b>Unit 2:</b> Limits and Continuity	<b>Duration:</b> 20 Days
<b>Standards/Learning Targets</b>	
<b>Focus Standards (Major Standards)</b>	
<p>N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities</p> <p>A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression</p> <p>A-APR.6 Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more</p>	

complicated examples, a computer algebra system.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

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| MP.2 | Reason abstractly and quantitatively.                            | <ul style="list-style-type: none"><li>● Make sense of quantities and their relationships in problems</li><li>● Learn to contextualize and de-contextualize</li><li>● Create coherent representations of problems</li></ul>   |
| MP.3 | Construct viable arguments and critique the reasoning of others. | <ul style="list-style-type: none"><li>● Understand and use information to construct arguments</li><li>● Make and explore the truth of conjectures</li><li>● Recognize and use counterexamples</li></ul>  |

MP.4	Model with Mathematics.	<ul style="list-style-type: none"> <li>● Justify conclusions and respond to arguments of others</li> <li>● Apply mathematics to problems in everyday life</li> <li>● Make assumptions and approximations</li> <li>● Identify quantities in a practical situation</li> <li>● Interpret results in the context of the situation and reflect on whether the results make sense</li> </ul>
MP.5	Use appropriate tools strategically.	<ul style="list-style-type: none"> <li>● Consider the available tools when solving problems</li> <li>● Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)</li> <li>● Make sound decisions of which of these tools might be helpful</li> </ul>
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MP.7	Look for and make use of structure.	<ul style="list-style-type: none"> <li>● Discern patterns and structures</li> <li>● Can step back for an overview and shift perspective</li> <li>● See complicated things as single objects or as being composed of several objects</li> </ul>
MP.8	Look for and express regularity in repeated reasoning.	<ul style="list-style-type: none"> <li>● Notice if calculations are repeated and look both for general methods and shortcuts</li> <li>● In solving problems, maintain oversight of the process while attending to detail</li> <li>● Evaluate the reasonableness of their immediate results is certain patterns and structures</li> </ul>

*Students will know...*

- The connection of pre-calculus to calculus through limits
- Continuous and Discontinuous functions and how they relate to limits
- The definition of limit
- When limits fail to exist
- How to use limit properties to evaluate limits
- How the squeeze theorem is derived and its application to trigonometric limits
- How continuity and limits are related
- How continuity and one-sided limits are related
- To find limits at infinity
- Use the epsilon delta definition of limit to find delta for a given epsilon

*Students will be able to..*

- Explain why the concept of limit was important in solving the tangent line and area problems
- Explain and write in mathematical terms the definition of limit.
- Show geometrically when a limit does not exist and give algebraic examples of those functions.
- State the relationship between continuity and limits vs discontinuity and limits
- Use graphs, numerical tables, and algebra methods to find limits
- Create a strategy which includes limit properties and methods to evaluate various limits.
- Explain the IVT and EVT theorems verbally and geometrically
- State how one-side limits help evaluate functions such as piecewise functions, step functions, and rational functions
- Find corresponding delta values for given epsilon values and can you relate this to real world applications

### Instructional Plan

**Suggested Activities**

**Resources**

- Graphing Calculator
- Microsoft Excel/PowerPoint

- Teacher-made tests, worksheets, warm-ups, and quizzes
- Computer software to support unit
- Smart board
- Document camera
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### Teacher Notes:

**Unit 3:** Derivatives

**Duration:** 60 Days

#### Standards/Learning Targets

#### Focus Standards (Major Standards)

N-RN.2 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the

expression.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-BF.1 Write a function that describes a relationship between two quantities.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

F-BF.1 Write a function that describes a relationship between two quantities.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-IF 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .

F-IF 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.

F-IF 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

### Supporting and Additional Standards

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| MP.5 | Use appropriate tools strategically.                             | <ul style="list-style-type: none"><li>● Consider the available tools when solving problems</li><li>● Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)</li><li>● Make sound decisions of which of these tools might be</li></ul> |

<p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>helpful</p> <ul style="list-style-type: none"> <li>• Communicate precisely to others</li> <li>• Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes</li> <li>• Calculate accurately and efficiently</li> </ul> <ul style="list-style-type: none"> <li>• Discern patterns and structures</li> <li>• Can step back for an overview and shift perspective</li> <li>• See complicated things as single objects or as being composed of several objects</li> </ul> <ul style="list-style-type: none"> <li>• Notice if calculations are repeated and look both for general methods and shortcuts</li> <li>• In solving problems, maintain oversight of the process while attending to detail</li> <li>• Evaluate the reasonableness of their immediate results is certain patterns and structures</li> </ul>
<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>• Derivatives of a function</li> <li>• Differentiability of a function</li> <li>• Rules for differentiation</li> <li>• Trigonometric derivatives</li> <li>• Implicit differentiation</li> <li>• Derivatives of Inverse Functions</li> <li>• Derivatives of Exponential and Logarithmic Functions</li> <li>• Whether a function is expressed explicitly or implicitly.</li> <li>• How to find a derivative using implicit differentiation.</li> <li>• When to use the appropriate differentiation rules as they implicitly differentiate.</li> <li>• The various methods for finding extrema on both a</li> </ul>	<p><i>Students will be able to..</i></p> <ul style="list-style-type: none"> <li>• Find the first derivative and higher order derivatives of any explicit or implicit function.</li> <li>• Recognize the structure of the function and understand what rules apply and in what order to perform the rules</li> <li>• Use the derivative to find a linear approximation.</li> <li>• Use the derivative to find rates of change for various application problems, including but not limited to physics and/or business</li> <li>• Distinguish between functions written in explicit form and implicit form.</li> <li>• Find the derivative of a function expressed implicitly.</li> <li>• Use the rules for differentiation when differentiating</li> </ul>

<p>closed interval and an open interval.</p> <ul style="list-style-type: none"> <li>● The relationship between position, velocity and acceleration.</li> <li>● The Mean value Theorem guarantees that functions increase/decrease depending on the value of the derivative.</li> <li>● The first and second derivatives indicate the behavior of the original function.</li> </ul>	<p>implicitly.</p> <ul style="list-style-type: none"> <li>● Set up and solve related rates problems.</li> <li>● Locate extrema on a closed interval.</li> <li>● Determine if Rolle's Theorem is applicable and if so, apply the theorem.</li> <li>● Determine if the Mean Value Theorem is applicable and if so, find the values guaranteed by the theorem.</li> <li>● Know the relationship between the position function, the velocity function and the acceleration function.</li> <li>● Use the Mean Value Theorem to determine the point at which the derivative equals the average rate of change.</li> <li>● Use the first and second derivative tests to determine maxima, points of inflection, intervals of increase/decrease, and intervals of concavity.</li> </ul>
<p><b>Instructional Plan</b></p>	
<p><b>Suggested Activities</b></p>	<p><b>Resources</b></p>
	<ul style="list-style-type: none"> <li>● Graphing Calculator</li> <li>● Microsoft Excel/PowerPoint</li> <li>● Teacher-made tests, worksheets, warm-ups, and quizzes</li> <li>● Computer software to support unit</li> <li>● Smart board</li> <li>● Document camera</li> <li>● <a href="http://www.ixl.com">www.ixl.com</a></li> <li>● <a href="http://www.purplemath.com">www.purplemath.com</a></li> <li>● <a href="http://www.brightstorm.com">www.brightstorm.com</a></li> </ul>

- [www.coolmath.com](http://www.coolmath.com)

## MODIFICATIONS

### *English Language Learners*

- Provide clear and specific directions
- Allow for alternate forms of responses- drawing or speaking instead of writing to demonstrate knowledge when you are not specifically assessing writing
- Provide class notes ahead of time to allow students to preview material and increase comprehension
- Provide extended time
- Model directions and provide gestures to increase understanding
- Simplify written and verbal instructions
- Allow the use of an online dictionary to look up the definition and hear the pronunciation of unknown words
- Create a nurturing environment with structured routines
- Teach study skills
- Gather materials such as visuals, models, manipulatives, videos and other tangible referents to contextualize the lesson.

### *Special Education*

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- Utilize graphic organizers

- Assign peer tutor
- Provide manipulatives
- Frequently check for understanding
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*Gifted and Talented*

- Extension activities
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- Technology Integration
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- Group Projects
- Tiered Activities

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**Core Instructional and Supplemental Materials**

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<b>Teacher Notes:</b>

<b>Unit 4: Integration</b>	<b>Duration: 72 Days</b>
<b>Standards/Learning Targets</b>	
<b>Focus Standards (Major Standards)</b>	
<p>N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> <p>N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p> <p>A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</p> <p>A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p> <p>A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p> <p>F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts;</p>	

intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-BF.1 Write a function that describes a relationship between two quantities.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.4 For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects

G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

### Supporting and Additional Standards

**The following Standards for Mathematical Practice and select New Jersey Student Learning Standards should be covered throughout the various units of the curriculum.**

## Standards for Mathematical Practices

MP.1	Make sense of problems and persevere in solving them	<ul style="list-style-type: none"><li>● Find meaning in problems</li><li>● Look for entry points</li><li>● Analyze, conjecture and plan solution pathways</li><li>● Monitor and adjust</li><li>● Verify answers</li><li>● Ask themselves the question: “Does this make sense?”</li></ul>
MP.2	Reason abstractly and quantitatively.	<ul style="list-style-type: none"><li>● Make sense of quantities and their relationships in problems</li><li>● Learn to contextualize and de-contextualize</li><li>● Create coherent representations of problems</li></ul>
MP.3	Construct viable arguments and critique the reasoning of others.	<ul style="list-style-type: none"><li>● Understand and use information to construct arguments</li><li>● Make and explore the truth of conjectures</li><li>● Recognize and use counterexamples</li><li>● Justify conclusions and respond to arguments of others</li></ul>
MP.4	Model with Mathematics.	<ul style="list-style-type: none"><li>● Apply mathematics to problems in everyday life</li><li>● Make assumptions and approximations</li><li>● Identify quantities in a practical situation</li><li>● Interpret results in the context of the situation and reflect on whether the results make sense</li></ul>
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<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>● The integral gives the area under the curve.</li> <li>● The rules for anti-differentiation.</li> <li>● The area under a curve is the amount of change of a function over time.</li> <li>● Slope fields represent the numeric derivative of a function at any point</li> <li>● There are various techniques needed for integration.</li> <li>● How to find displacement and/or total distance from a given velocity.</li> <li>● How to find boundaries for integrating between intersecting curves.</li> <li>● When to use horizontal or vertical rectangles for finding area based upon given conditions.</li> <li>● Compute the areas and volumes manually and through</li> </ul>	<p><i>Students will be able to..</i></p> <ul style="list-style-type: none"> <li>● Evaluate an integral, definite or indefinite.</li> <li>● Apply the Fundamental Theorem of Calculus</li> <li>● Use integration by substitution and integration by parts as needed to evaluate integrals</li> <li>● Sketch a slope field</li> <li>● Match a differential equation with its slope field.</li> <li>● Solve a separable differential equation using substitution and partial fraction decomposition.</li> <li>● Use Euler's Method to solve an initial value differential value at a given point.</li> <li>● Find displacement an object at a given moment.</li> <li>● Find the total distance traveled over an given interval of time.</li> <li>● Use algebraic methods or available technology to find</li> </ul>

the use of available technology.

point(s) of intersection that will be the boundaries of bounded regions.

- Use algebraic methods or available technology to find areas and volumes.
- Use dimensional analysis to determine what quantities and units of measure are calculated. *For example – the calculated area under velocity curve (ft/sec) for a given time interval (in sec.) would result in how far (ft.) the object travels.*

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#### Suggested Activities

#### Resources

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